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SIMBAT THEORETICAL MANUAL

ABSTRACT This manual develops the theory of the Conditional Ocean Wave Simulation Model (SIMBAT). SIMBAT is a high performance FORTRAN F77 based computer model that uses the Fast Fourier Transform (FFT) to produce ocean wave properties for ocean engineering applications. The model assumes the wave properties form a Gaussian stochastic process. SIMBAT may be used to perform a conditional or unconditional simulation. A conditional simulation uses a measured or existing input time series and forces the simulated wave properties to adhere to the input time series but also follow the laws of multivariate normal probability. An unconditional simulation uses a measured or created ocean wave spectra to randomly simulate the ocean wave properties. Program features include: creation of directional ocean wave spectra, water particle kinematic stretching, conditioning time series using an input time series less than or equal to the simulation length, use of Legendre orthogonal polynomials for post-creation of wave properties by program CKPOLY, and error checking.

NAVAL CIVIL ENGINEERING LABORATORY PORT HUENEME CALIFORNIA 93043-4328

METRIC CONVERSION FACTORS

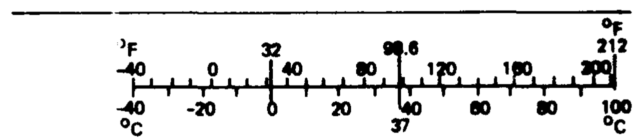
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2,000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
^oF	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	^oC

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1,000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
^oC	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	^oF



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This manual develops the theory of the Conditional Ocean Wave Simulation Model (SIMBAT). SIMBAT is a high performance FORTRAN F77 based computer model that uses the Fast Fourier Transform (FFT) to produce ocean wave properties for ocean engineering applications. The model assumes the wave properties form a Gaussian stochastic process. SIMBAT may be used to perform a conditional or unconditional simulation. A conditional simulation uses a measured or existing input time series and forces the simulated wave properties to adhere to the input time series but also follow the laws of multivariate normal probability. An unconditional simulation uses a measured or created ocean wave spectra to randomly simulate the ocean wave properties. Program features include: creation of directional ocean wave spectra, water particle kinematic stretching, conditioning time series using an input time series less than or equal to the simulation length, use of Legendre orthogonal polynomials for post-creation of wave properties by program CKPOLY, and error checking.

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EXECUTIVE SUMMARY

This report provides a theoretical description of the Conditional Ocean Wave Simulation Model called SIMBAT. The mathematical foundations, probability theory, and numerical methods employed are explained. This report provides the user with the necessary theoretical information to utilize SIMBAT in an educated manner. Along with this document, a supplementary document, TN-1838 SIMBAT User's Manual (Borgman, et al., 1991), which provides information for use of the software, is available from the Naval Civil Engineering Laboratory (NCEL). SIMBAT was developed by Dr. L. E. Borgman at the University of Wyoming for NCEL

The primary objective of the SIMBAT development was to provide an efficient means to assist in analyzing the dynamic motions of deepwater semisubmersible platforms to be used for Offshore Tactical Aircrew Combat Training Facilities (Shields, et al., 1987). This development was funded by the Office of Naval Technology (ONT) under the Navy Exploratory Development Program.

SIMBAT development focused on the simulation of water wave properties to expedite subsequent calculation of wave loading. For example, if the Morison equation (Sarpkaya and Isaacson, 1981) is used to compute the wave loads on a slender member, the water particle velocities and accelerations will be required as input for each time step and at each load point across the structure. SIMBAT produces this data by use of fast frequency domain methods and by approximation with Legendre orthogonal polynomials (Hochstrasser, 1964).

SIMBAT provides the option to create either conditionally or unconditionally simulated water particle kinematics. A conditional simulation utilizes a measured ocean wave spectrum or measured time series history to "condition" the simulation to create associated water particle kinematics that adhere to the properties of the input data but also follow the laws of normal probability theory. This is particularly useful if the user wants to impose a large measured wave profile on the structure and create the associated kinematics for that profile to determine the extreme loads on the structure. The advantage of this option is that a large wave profile is certain to occur in the computer simulation where ordinarily a much longer time domain simulation would be required before an extreme wave profile would be realizable. Thus, the design engineer may utilize SIMBAT to produce the appropriate water particle kinematics for extreme wave loading in a reasonable amount of computer time.

The unconditional simulation will produce wave properties in accordance with an input ocean wave spectrum and that follow a multivariate normal probability law. The wave properties are "unconditionally" simulated randomly by an input seed number specified by the user.

Current procedures available for simulating ocean wave properties are based on time domain superposition and filtering of Gaussian white noise. Substantial savings in computer simulation time and expenses can be realized if the wave properties are simulated in the frequency domain as is done in SIMBAT.

One of the major features of the simulation model is its ability to produce wave properties over a large three-dimensional spatial region using a Legendre polynomial fit. The wave load points on the offshore structure are calculated at each time step in an expedient manner as opposed to the standard industry method of computing the wave properties in the time domain

at each load point. Orders of magnitude of computer time may be saved as a result of this method. If SIMBAT is used for this application, the design engineer would merge the SIMBAT post processor, CKPOLY, with a structural analysis package to read the water particle kinematics from the SIMBAT output files.

SIMBAT can be used for the following applications:

(a) Determination of ocean wave properties for computing wave loading on offshore structures that will be used in connection with dynamic response simulation models.

(b) Wave force studies in random waves.

(c) Creation of random directional seas for design applications.

In addition to these three applications, SIMBAT may also be used for the conditional simulation of waves in a model test basin (e.g., the creation of a large wave train or wave groups in a basin that would adhere to the input wave autospectra and retain the statistical properties of the multivariate normal probability law). This would allow the experimentalist to perform tests in a shorter period of time.

CONTENTS

CONTENTS

OBJECTIVE

This report provides a theoretical description of the Conditional Ocean Wave Simulation Computer Model (SIMBAT). This report supplements the SIMBAT User's Manual (Borgman et al., 1991), which provides the instructions for use of the SIMBAT software.

INTRODUCTION

Highly regular, single direction, single frequency wave trains are quite rare in the ocean. The predominant wave systems present are irregular, with at least some degree of short crestedness (Figure 1). The wave conditions in an intense storm are particularly chaotic and turbulent, yet these are the conditions most significant to the engineer designing a structure for emplacement in the ocean.

Several options are available to the designer. Nonlinear, unidirectional waves of permanent form, such as those produced by Stokes' theory or various numerical solutions, can be used for each large "bump" of sea surface. This produces a conservative or maximal estimate of the water particle velocities and accelerations, because each component of the wave form reinforces the others to push and pull together. Alternatively, a linear superposition of many independent components, each with its own direction, phase, and amplitude, can be used to obtain the wave kinematics.

Both of these options are imperfect models of the real ocean. The fault of the nonlinear, unidirectional theory is that it imposes an artificial regularity on its estimates of the wave kinematics. The theory based on linear superposition allows the irregularity, but in turn produces a linear solution to wave conditions that are known to have skewed nonlinear characteristics of sharper crests and flatter troughs. The design engineer must judge which approach has the least imperfection for the application intended. There are situations where each is the most appropriate compromise. A discussion of these options and a comparison with field data are given by Forristall, Ward, Borgman, and Cardone (1978).

More to the point, the theory used for the wave kinematics must be evaluated in tandem with the wave force formula and the values of the force coefficients selected. An overestimation within the wave kinematics algorithm can be somewhat compensated for with lower force coefficients, and vice versa.

The Morison equation is generally accepted as a satisfactory approximation for wave force, although other alternatives and variations have been investigated from time to time. One common modification is to vary the drag coefficient with time according to some locally evaluated Reynolds number.

The particular combination of wave theory and force formula selected for use really needs to be calibrated against measured wave data under real oceanic conditions. Most oil companies have gone through this exercise at least once for the schemes they have found useful in design. The larger companies proceed with such evaluations almost continually to match new oceanic conditions as they arise in the company's operations. There is, of course, some diversity in computational procedures from company to company, because each may choose to adjust the overall kinematics and force calculations in different places to calibrate the produced estimates to agree with measured data.

Various reviews of nonlinear wave theory (Dean, 1970; Sarpkaya and Isaacson, 1981), random wave theory (Borgman, 1972; Ochi, 1982) and force formulas (Dean and Borgman, 1986; Sarpkaya and Isaacson, 1981) have been published. Dean and Borgman (1986, p. 341) also give a good summary of field measurement programs. It does not appear useful to provide yet another summary of these topics. Instead, various new results largely related to the estimation of kinematics in irregular wave fields as input to force computations will be presented. Computer simulation of wave kinematics and the special topic of conditional simulation will be particularly emphasized. The inclusion of the word "forces" in the chapter title is intended to indicate that the formulas for kinematics are directed toward providing a basis for wave force computations. New public domain computer software is just now becoming available that enhances the usefulness of these techniques in engineering design. Sources for such software will be briefly reviewed.

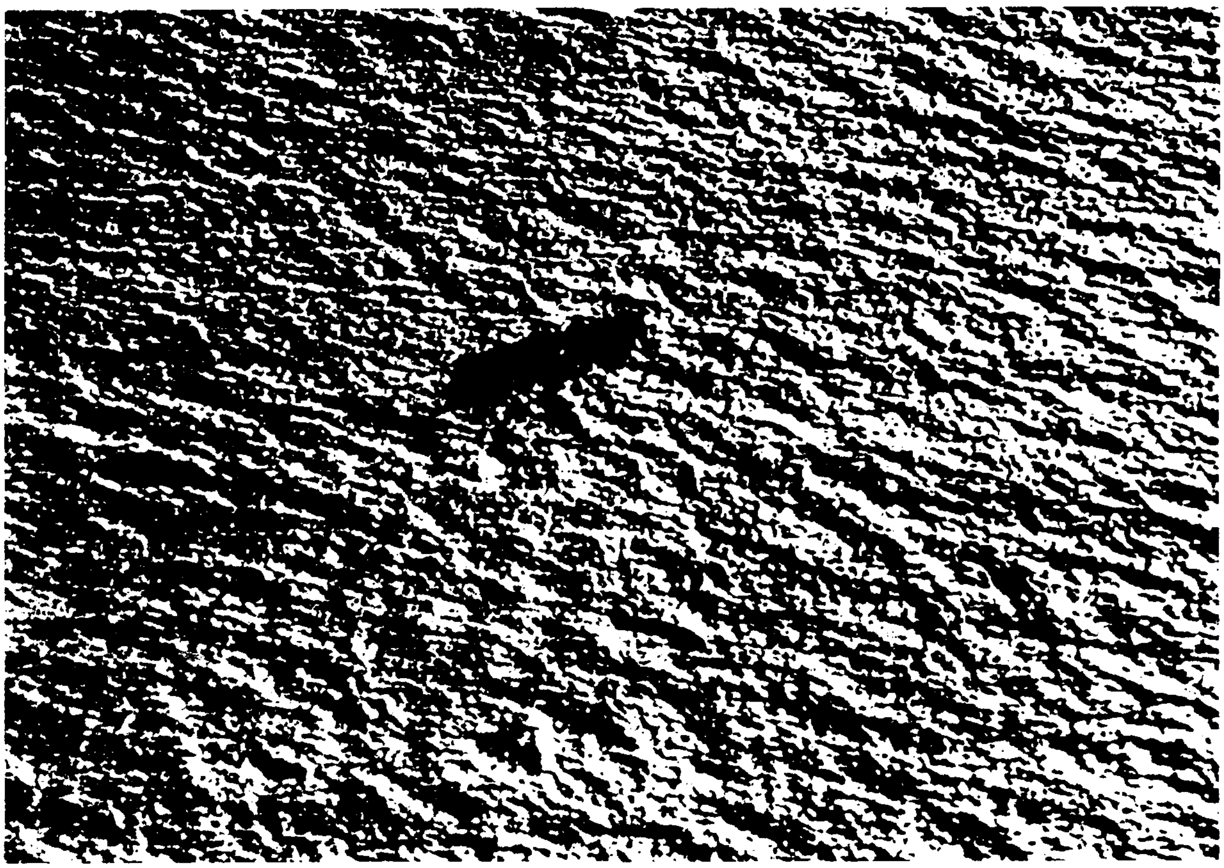

Figure 1
Aerial photograph of an irregular sea surface (ship in photo shows scale) (from Pierson, Neumann, and James, 1990).

BACKGROUND

The Naval Civil Engineering Laboratory (NCEL), under the Offshore Aircrew Combat Training System task of the Navy Exploratory Development Program, was tasked to develop technology necessary to design and construct reliable and cost effective unmanned ocean platforms for deepwater (600 to 10,000 feet) Tactial Aircrew Combat Training System (TACTS) applications (Shields, et al., 1987). One of the objectives of this development program was to develop, modify, integrate, and validate computer models for simulation of deepwater moored platform motions. Investigations of various candidate platforms resulted in the semisubmersible concept being selected as the most suitable platform type (Shields, et al., 1983).

The SEASTAR (PMB Engineering, 1990) nonlinear structural finite element analysis program was selected to model the hydrodynamic loads and dynamic response of the semisubmersible/mooring system. SIMBAT was developed to meet the following requirements:

1. Wave force studies in random seas.

2. Wave property input in dynamic response simulation model, SEASTAR.

3. Random directional seas representation for design applications.

In order to gain an insight into the dynamic response of deepwater TACTS semisubmersible platforms, the dynamic response simulation model SEASTAR was used. The preferred environmental data base for SEASTAR, when used in design analyses, consists of a number of actual ocean wave time series corresponding to major storms (i.e., 50, 75, 100 year storm) or other environmental scenarios of interest (i.e., those for fatigue analyses or operational sea states) at the sites of interest. Since these data are unavailable in most design projects, simulated ocean wave property data should be used.

Current procedures available for simulating ocean wave properties are based on time domain superposition and filtering of Gaussian white noise. Substantial savings in simulation time and expenses can be realized if the discrete Fourier transform of the desired time histories of the wave properties are simulated directly in the frequency domain. The frequency domain simulation methods are ten to a hundred times faster than the time domain simulation techniques. In addition, the computationally very rapid Fast Fourier Transform (FFT) can be used to revert the simulation to the time domain.

All simulation schemes are based on the introduction of random numbers. The engineer/oceanographer/mathematician builds in the wave properties (design wave/wave grouping) to be preserved. If one has an actual sequence of ocean wave data and wishes to study the motion response associated with that particular sequence, currently available simulation procedures will not accommodate these data. Conditional simulations are necessary, with the randomness being restricted so as to produce ocean wave simulation data, conditional on the wave having the required values. These required values may, for example, be a series of wave heights expected in a typical storm or other environmental scenarios at the site.

Conditional simulation of ocean wave properties is a very recent research topic with wide application to Navy projects. Techniques of conditional simulation have been used in geological problems and are now applied to ocean wave applications as used in SIMBAT.

During the development of SIMBAT, the use of Legendre orthogonal polynomials (Hochstrasser, 1964) was added to SIMBAT to create three-dimensional wave properties over a large spatial region. The use of these polynomials along with the frequency domain method of creating the wave properties was determined to provide a computationally much faster algorithm than computing the wave properties at all grid points over all time. This application utilizing Legendre polynomials is now considered one of the major beneficial aspects of SIMBAT, especially for compliant systems.

OVERVIEW OF SIMBAT MODEL AND SOFTWARE

SIMBAT is a computer model that calculates ocean wave properties, such as water particle kinematics for use in various ocean engineering applications. The model assumes the wave properties form a Gaussian stochastic process. This means that all wave properties at any selected set of times and locations follow a multivariate normal probability law.

The SIMBAT software can create the Ochi-Hubble, Pierson-Moskowitz, Bretschneider, JONSWAP, and Wallops Spectral Models or read in the user's own spectra, if desired. The waves may be directionally spread by either wrapped normal, cosine-squared, or von Mises methods. There are options to stretch wave properties above mean water level using Rodenbusch-Forristall delta stretch, Reid-Wheeler stretch, truncation extrapolation, functional extrapolation, linear extrapolation, and gamma extrapolation.

SIMBAT has the option to perform a single simulation of wave properties at one or more locations in space using frequency domain methods directly or using the application of Legendre orthogonal polynomials. If the polynomials are used, a separate program, CKPOLY, is used to create the wave properties from SIMBAT output. CKPOLY is provided with SIMBAT but is separate to allow the user to embed the program in their structural analysis software.

For conditional simulation, the user can provide SIMBAT with a measured large wave profile, wave groups, or any other wave properties that are presentative of the environmental area of interest. SIMBAT will constrain the simulated wave properties to follow the measured data while also maintaining the statistical laws appropriate for ocean waves.

The latest version of SIMBAT, Release 3.0, now contains accuracy verification algorithms to compare kinematics from the different simulation methods in SIMBAT.

SIMBAT has the option to write ASCII output data files as it proceeds through the simulation so that the user may verify the integrity of the data. Therefore, if SIMBAT is executed in a multitasking, multiwindow environment, the user can confirm the simulation is working properly on his/her computer throughout each level of the simulation.

This study was motivated by the need for computer-efficient calculation procedures and computer codes for the preparation of accurate ocean wave kinematics and properties for multiple-frequency, multiple-direction seas. The immediate application was the design of moored, floating instrument towers for Navy gunnery ranges. However, the extensive theory and efficient computational algorithms should prove useful in many other ocean engineering applications.

THEORETICAL FORMULATIONS OF THE SIMBAT MODEL

Coordinate System Specifications

The ocean wave kinematics will be referenced to a general horizontal coordinate system. All horizontal coordinate axes are established within navigation headings measured clockwise from true north:

$$\begin{aligned} \theta_x &= \text{direction of positive x axis} \\ \theta_y &= \text{direction of positive y axis} \\ |\theta_x - \theta_y| &= 90^\circ \end{aligned} \tag{1}$$

Let the vertical axis z be zero at mean water level and positive downward.

The direction of travel of a wave is θ in navigation heading. The wave is traveling toward direction θ if $\beta = 1$ and is coming from direction θ if $\beta = -1$.

Basic Wave Properties

Eight wave properties are of interest. In terms of real functions, these are:

1. The water level elevation,

$$\eta(x,y,t) = a \cos\{\beta_0 k[x \cos(\theta - \theta_x) + y \cos(\theta - \theta_y)] - 2\pi f t - \phi\} \tag{2}$$

2. The components of water particle velocity,

$$\begin{bmatrix} V_x(x,y,z,t) \\ V_y(x,y,z,t) \end{bmatrix} = a(2\pi f) \frac{\cosh[k(d - z)]}{\sinh(k d)} \begin{bmatrix} \beta_0 \cos(\theta - \theta_x) \\ \beta_0 \cos(\theta - \theta_y) \end{bmatrix}$$

$$* \cos[\beta_0 k\{x \cos(\theta - \theta_x) + y \cos(\theta - \theta_y)\} - 2\pi f t - \phi] \tag{3}$$

$$V_z(x,y,z,t) = a(2\pi f) \frac{\sinh[k(d - z)]}{\sinh(k d)}$$

$$* \sin[\beta_0 k\{x \cos(\theta - \theta_x) + y \cos(\theta - \theta_y)\} - 2\pi f t - \phi] \tag{4}$$

3. The components of water particle acceleration,

$$\begin{bmatrix} a_x(x,y,z,t) \\ a_y(x,y,z,t) \end{bmatrix} = a\,(2\,\pi\,f)^2\,\frac{\cosh[k(d-z)]}{\sinh(k\,d)} \begin{bmatrix} \beta_0 \cos(\theta-\theta_x) \\ \beta_0 \cos(\theta-\theta_y) \end{bmatrix}$$

$$* \sin[\beta_0 k\,\{x\cos(\theta-\theta_x) + y\cos(\theta-\theta_y)\} - 2\,\pi\,f\,t - \phi] \quad (5)$$

$$a_z(x,y,z,t) = -\,a(2\,\pi\,f)^2\,\frac{\sinh[k(d-z)]}{\sinh(k\,d)}$$

$$* \cos[\beta_0 k\,\{x\cos(\theta-\theta_x) + y\cos(\theta-\theta_y)\} - 2\,\pi\,f\,t - \phi] \quad (6)$$

4. The water pressure anomaly (plus and minus about hydrostatic pressure) divided by ρg,

$$\frac{p\,(x,u,z,t)}{\rho\,g} = a\,\frac{\cosh[k\,(d-z)]}{\cosh(k\,d)}$$

$$* \cos[\beta_0 k\,\{x\cos(\theta-\theta_x) + y\cos(\theta-\theta_y)\} - 2\,\pi\,f\,t - \phi] \quad (7)$$

In these formulas:

a = wave amplitude
f = wave frequency
d = water depth
k = wave number = 2π/wave length
ϕ = wave phase
ρ = water density
g = acceleration due to gravity

and it is assumed that wave number and wave frequency are related by the formula:

$$(2\,\pi\,f)^2 = g\,k\,\tanh(k\,d) \quad (8)$$

Wave Properties in Complex Form

Through the use of the complex form and $\cos\alpha$ and $\sin\alpha$, where

$$\cos\alpha = \frac{\exp(i\,\alpha) + \exp(-\,i\,\alpha)}{2} \quad (9)$$

$$\sin\alpha = \frac{\exp(i\alpha) - \exp(-i\alpha)}{2i} \tag{10}$$

All of the wave properties listed above can be expressed in the form:

$$b(x,y,z,t) = \frac{a e^{i\phi}}{2} G(z)\, T(f)\, H(\theta) \exp[-i\,\beta_0 k \{x \cos(\theta - \theta_x) + y \cos(\theta - \theta_y)\}] \exp(i\, 2\pi f t) \tag{11}$$

for positive f. The original real-valued wave time property equals $b(f) + b(f)^*$, where

$$b*(f) = \text{complex conjugate of } b(f) \tag{12}$$

The functions G, T, and H for each wave property are:

1. Water level elevation,

$$G(z) = T(f) = H(\theta) \equiv 1.0 \tag{13}$$

2. Velocity,

$$G(z) = \begin{cases} \dfrac{\cosh[k(d-z)]}{\sinh(kd)} = \dfrac{\{e^{-kz} + e^{-k(2d-z)}\}}{\{1 - e^{-2kd}\}}, & \text{for } V_x \text{ and } V_y \\[2ex] \dfrac{\sinh[k(d-z)]}{\sinh(kd)} = \dfrac{\{e^{-kz} - e^{-k(2d-z)}\}}{\{1 - e^{-2kd}\}}, & \text{for } V_z \end{cases} \tag{14}$$

$$T(f) = \begin{cases} 2\pi f, & \text{for } V_x \text{ and } V_y \\ 2\pi f i, & \text{for } V_z \end{cases} \tag{15}$$

$$H(\theta) = \begin{cases} \beta_o \cos(\theta - \theta_x), & \text{for } V_x \\ \beta_o \cos(\theta - \theta_y), & \text{for } V_y \\ 1.0, & \text{for } V_z \end{cases} \tag{16}$$

3. Acceleration,

$$G(z) = \begin{cases} \dfrac{\cosh[k(d-z)]}{\sinh(kd)} = \dfrac{\{e^{-kz} + e^{-k(2d-z)}\}}{\{1 - e^{-2kd}\}}, & \text{for } a_x \text{ and } a_y \\[2ex] \dfrac{\sinh[k(d-z)]}{\sinh(kd)} = \dfrac{\{e^{-kz} - e^{-k(2d-z)}\}}{\{1 - e^{-2kd}\}}, & \text{for } a_z \end{cases} \tag{17}$$

$$T(f) = \begin{cases} (2\pi f)^2 i, & \text{for } a_x \text{ and } a_y \\ -(2\pi f)^2, & \text{for } a_z \end{cases} \tag{18}$$

$$H(\theta) = \begin{cases} \beta_0 \cos(\theta - \theta_x), & \text{for } a_x \\ \beta_0 \cos(\theta - \theta_y), & \text{for } a_y \\ 1.0, & \text{for } a_z \end{cases} \tag{19}$$

4. Pressure anomaly,

$$G(z) = \frac{\cosh[k(d-z)]}{\cosh(kd)} = \frac{\{e^{-kz} + e^{-k(2d-z)}\}}{\{1 + e^{-2kd}\}} \tag{20}$$

$$T(f) = 1.0 \tag{21}$$

$$H(\theta) = 1.0 \tag{22}$$

In the above definitions, the hyperbolic function terms involving the vertical coordinate z and water depth d have been expressed in algebraically equivalent forms, which makes it easy to extend the formulas to very large water depths. These forms also have advantages later in the sections on the treatment of compliant structures.

Irregular Waves by Superposition

An irregular wave train can be obtained by the summation of many different simple wave forms. For example, the general formulas in Equation 11 can be summed to obtain, with some obvious extensions of notation,

$$b(t;x,y,z) = \sum_{m=-M}^{M} \sum_{j=1}^{J} a_{mj} e^{i\phi_{mj}} G(f_m,z)\, T(f_m)\, H(\theta_j) \tag{23}$$
$$* \exp[-i\, \beta_0\, k_m \{x \cos(\theta_j - \theta_x) + y \cos(\theta_j - \theta_y)\}] \exp(i\, 2\, \pi\, f_m\, t)$$

The summation extends over an arbitrarily selected list of frequencies $\{f_m;\ m = 1,2,3,\ldots,M\}$, and a corresponding list of directions $\{\theta_j;\ j = 1,2,3,\ldots,J\}$. By convention,

$$f_0 = 0, \qquad f_{-m} = -f_m \tag{24}$$

and the terms in the summation for negative frequencies are taken to be the complex conjugates of those for the corresponding positive frequencies. This forces the wave property time series to be real-valued.

Irregular Waves in Frequency Domain

Let

$$B(f_m;x,y,z) = \sum_{j=1}^{J} a_{mj} e^{i\phi_{mj}} G(f_m,z)\, T(f_m)\, H(\theta_j) \tag{25}$$
$$* \exp[-i\, \beta_0\, k_m \{x \cos(\theta_j - \theta_x) + y \cos(\theta_j - \theta_y)\}]$$

Equation 23 may be expressed as a discrete Fourier transform,

$$b(t;x,y,z) = \sum_{m=-M}^{M} B(f_m;x,y,z) \exp(i\, 2\, \pi\, f_m\, t) \tag{26}$$

This form is particularly convenient since it permits, with some slight modifications, the high-speed computation of wave property time series with the Fast Fourier Transform (FFT) algorithm.

Fast Fourier Transform (FFT) Algorithm

The FFT algorithm provides a very efficient and rapid procedure for calculating the two formulas (Blahut, 1985):

$$W_m = \frac{1}{N} \sum_{n=0}^{N-1} w_n \exp(-i\, 2 \pi\, m\, n/N) \tag{27}$$

$$w_n = \sum_{m=0}^{N-1} W_m \exp(i\, 2 \pi\, m\, n/N) \tag{28}$$

for $m = 0,1,2,\ldots,N-1$ and $n = 0,1,2,\ldots,N-1$. These are exact discrete transformations for computing one of the two sequences, $\{W_m;\ m = 0,1,2,\ldots,N-1\}$ and $\{w_n;\ n = 0,1,2,\ldots,N-1\}$, from the other.

Some properties of the FFT formulas deserve mention. Because

$$\exp(\pm i\, 2 \pi\, m\, n/N) = \cos(2 \pi\, m\, n/N) \pm i \sin(2 \pi\, m\, n/N) \tag{29}$$

both summations are formally periodic, with period N. This means that both sequences $\{W_m\}$ and $\{W_n\}$ are forced by their mathematics to have this periodicity. Also, both sequences are complex-valued, in general. However, in the applications here, the w_n are real-valued and the W_m are complex-valued. A necessary and sufficient condition for the w_n to be real-valued is that:

$$W_m = W^*_{N-m} \tag{30}$$

for $0 \le m \le N/2$.

Wave Properties in the FFT Format

The conversion of Equation 23 to the FFT format is achieved by discretizing time and frequency,

$$\begin{aligned} t &= n\,\Delta t, & n &= 0,1,2,\ldots,N-1 \\ f_m &= m\,\Delta f, & m &= 0,1,2,\ldots,N/2 \end{aligned} \tag{31}$$

where the time and frequency increments are taken to satisfy

$$(\Delta t)(\Delta f) = 1/N \tag{32}$$

With this constraint, the argument in the complex exponential in Equation 25 simplifies to:

$$2 \pi f_m t = 2 \pi\, m\, n/N \tag{33}$$

In addition, the periodicity implicit in the FFT formulas leads naturally to the translation of the negative frequency values in Equation 26 to the positive frequencies:

$$B(\{N-m\}\Delta f;x,y,z) = B(-m\Delta f;x,y,z) \qquad (34)$$

The complex-valued, skew conjugate symmetry imposed on Equation 23 then becomes the FFT symmetry in Equation 30 that forces the transform to be real-valued.

With these modifications, the general wave property formulas in Equation 26 become:

$$b(n\Delta t;x,y,z) = \sum_{m=0}^{N-1} B(m\Delta f;x,y,z)\exp(i2\pi mn/N) \qquad (35)$$

The values of N and Δf should be selected so that all the energetic frequencies of importance are enclosed within the bounds:

$$0 < m_B\Delta f < f < m_L\Delta f << 1/(2\Delta t) \qquad (36)$$

The interval (m_B,m_L) ordinarily extends over, perhaps, 150 Δf increments out of $N = 2048$ terms in the sequence. The other FFT coefficients can be set to zero, except for those determined by complex conjugation in symmetry about $m = N/2$.

In summary, a frequency domain computation of the wave property time series consists of the following steps:

1. $B(m\Delta f;\ x,\ y,\ z)$ is computed from Equation 25 for $m_B \le m \le m_L$.

2. By conjugate symmetry, for $m_B \le m \le m_L$,

$$B(\{N-m\}\Delta f;x,y,z) = B^*(m\Delta f;x,y,z) \qquad (37)$$

3. The rest of the coefficients are set to zero.

4. The sequence of $B(m\Delta f;\ x,\ y,\ z)$ values are transformed with the Fast Fourier Transform algorithm by:

$$b(n\Delta t;x,y,z) = \sum_{m=0}^{N-1} B(m\Delta f;x,y,z)\exp(i2\pi mn/N) \qquad (38)$$

This total process yields the full time series for $n = 0,1,2,\ldots,N-1$.

THE COMPLEX-VALUED AMPLITUDE MATRIX

The irregular train and all its associated linear properties, such as its kinematics, are specified completely, for the FFT approach, by:

$$A_{mj} = a_{mj} e^{i\phi_{mj}} \tag{39}$$

for $m_B \leq m \leq m_L$ and $j = 1,2,\ldots,J$. The amplitudes A_{mj} can be arranged into a complex matrix with rows specifying direction and columns designating frequencies. The other functions in Equation 23 are transfer functions that convert the amplitudes into the various wave properties.

Any procedure, either deterministic or random, that generates the complex amplitude matrix fully determines all the wave property time series. Much of the following discussion will be concerned with various choices and their consequences related to the probabilistic selection of values for the complex amplitudes.

Characterization of Irregular Waves

The directional spectral density $S(f,\theta)$ is the function most commonly used to characterize the mean-square oscillation of the wave components at different frequencies and directions. More specifically, define the two-sided spectrum as:

$$\begin{aligned} 2\,S(f_m,\theta_j)\,\Delta f\,\Delta\theta &= a_{mj}^2/2 \\ &= \text{mean-square oscillation of a cosine} \\ &\quad\ \text{wave with amplitude } a_{mj} \end{aligned} \tag{40}$$

The mean-square oscillation of the sea surface in the irregular wave train, if all components behave independently, becomes:

$$\sum_{m=1}^{M} \sum_{j=1}^{J} \frac{a_{mj}^2}{2} = 2 \int_0^{\infty} \int_0^{2\pi} S(f,\theta)\, d\theta\, df \tag{41}$$

In the case where the sea surface is taken as a random process, this becomes:

$$\mathrm{var}(\eta) = 2 \int_0^{\infty} \int_0^{2\pi} S(f,\theta)\, d\theta\, df \tag{42}$$

A function of angle θ at each frequency, called the spreading function, is defined as:

$$D(\theta,f) = S(f,\theta)/S(f) \tag{43}$$

where

$$S(f) = \int_0^{2\pi} S(f,\theta)\, d\theta \tag{44}$$

The relation in Equation 43 can also be written as:

$$S(f,\theta) = S(f)\, D(\theta;f) \tag{45}$$

Many formulas have been proposed and studied as models for S(f) and the spreading function (Borgman, 1979; Ochi, 1982). The most effort has been spent seeking the best characterization for fully arisen seas, in order to improve wave forecasts and hindcasts. The emphasis for structural design is somewhat different, where it is desirable to be able to model a wide variety of sea conditions, from very narrow-banded to very broad-banded, and from unidirectional conditions out to broadly spread seas. Of particular importance are seas that are multimodal in direction and/or frequency.

All of the spreading functions commonly used (the cosine-squared, the von Mises, and the wrapped normal formulas) have almost exactly the same shape. That is, parameters can be selected that make the formulas very nearly lie on top of each other. All of these are unimodal and symmetric about the principal direction.

There are several very good discussions of these various choices of formulas (Sarpkaya and Isaacson, 1981, pp. 504-520; Ochi, 1982, pp. 308-346; and Muga, 1984, pp. 163-175). Rather than repeating still another summary here, an often overlooked specific choice that has many advantages in engineering studies will be introduced. This choice consists of the Ochi-Hubble spectrum, as combined with the wrapped normal spreading function. Superpositions of such spectral models can be used to obtain multimodal conditions.

Ocean Spectral Models

Ochi-Hubble Spectra Formula. The formulation developed by Ochi and Hubble (1976) was expressed as a one-sided spectrum with radian frequency. For consistency with the present treatment, this is changed to a two-sided, cycles-per-second formula,

$$S(f) = \frac{2\,[(4\,\lambda + 1)\, f_0^4/4]^{\lambda}\, \sigma^2 \exp[-(4\,\lambda + 1)\,(f_0/f)^4/4]}{\Gamma(\lambda)\, |f|^{4\lambda+1}} \tag{46}$$

where $\Gamma(\lambda)$ is the complete gamma function. It is straightforward to show by calculus that S(f) integrates over $(-\infty, \infty)$ to obtain σ^2. The change of variable,

$$y = (4\lambda + 1)(f_0/f)^4/4 \tag{47}$$

is made within the integral to reduce the expression to:

$$\text{variance} = 2\int_0^\infty S(f)\,df = \sigma^2[\Gamma(\lambda)]^{-1}\int_0^\infty e^{-y}\,y^{\lambda-1}\,dy \tag{48}$$

If S(f) is differentiated with respect to f and set equal to zero, to determine the value at which S(f) is a maximum, the result is $f = f_0$. The maximum value for the spectrum, S_0, is then:

$$S_0 = \frac{2[(4\lambda + 1)/4]^\lambda}{\Gamma(\lambda)\,f_0}\,\sigma^2 \exp[-(4\lambda + 1)/4] \tag{49}$$

From this,

$$P = \frac{S_0 f_0}{\sigma^2} = \frac{2[(4\lambda + 1)/4]^\lambda \exp[-(4\lambda + 1)/4]}{\Gamma(\lambda)} = G(\lambda) \tag{50}$$

where $G(\lambda)$ is the function of λ and P the parameter so defined.

The effective width δ of a spectral density will be defined as the width of a square pulse with the same height, S_0, and area, $\sigma^2/2$, as the spectral density from $(0, \infty)$. That is,

$$\delta S_0 = \frac{\sigma^2}{2} \tag{51}$$

$$\delta = \frac{\sigma^2}{2\,S_0} \tag{52}$$

Another expression for the parameter P is:

$$P = \frac{f_0}{2\,\delta} = G(\lambda) \tag{53}$$

This conveniently relates λ to the effective width and peak frequency of the spectrum formulas.

If $\lambda = 1$, the Ochi-Hubble spectrum reduces to the usual Pierson-Moskowitz-Bretschneider formulas. If $\lambda > 1$, say 100 or so, the spectrum is very narrow-banded. If $\lambda < 1$, the spectrum is broader than the usual spectral models. Table 1 shows P versus λ. It is easy to generate more elaborate tables from Equation 50 using a computer, or to solve by Newton-Raphson iteration (Press, et al., 1986, pp. 254-259) for the value of λ required to achieve a give P value.

Table 1
Lambda Versus Spectral Parameters in the Ochi-Hubble Formula

Lambda/ Multiplier	Multiplier				
	0.01	0.1	1.0	10.0	100.0
1.0	0.0153	0.133	0.716	2.49	7.97
1.1	0.0168	0.145	0.758	2.62	8.36
1.2	0.0183	0.156	0.798	2.74	8.73
1.3	0.0198	0.167	0.836	2.85	9.09
1.4	0.0213	0.177	0.873	2.96	9.43
1.5	0.0228	0.188	0.908	3.07	9.77
1.6	0.0242	0.198	0.942	3.17	10.09
1.7	0.0257	0.208	0.975	3.27	10.40
1.8	0.0272	0.218	0.006	3.336	10.70
1.9	0.0286	0.227	1.037	3.46	10.99
2.0	0.0301	0.237	1.067	3.55	11.28
2.1	0.0315	0.246	1.096	3.64	11.56
2.2	0.0330	0.255	1.125	3.72	11.83
2.3	0.0344	0.264	1.153	3.81	12.09
2.4	0.0359	0.273	1.180	3.89	12.35
2.5	0.0373	0.281	1.206	3.97	12.61
2.6	0.0387	0.290	1.232	4.05	12.86
2.7	0.0401	0.298	1.258	4.13	13.10
2.8	0.0416	0.306	1.283	4.20	13.35
2.9	0.0430	0.314	1.307	4.28	13.58
3.0	0.0444	0.322	1.331	4.35	13.81
3.2	0.0472	0.338	1.378	4.50	14.27
3.4	0.0500	0.353	1.423	4.64	14.71
3.6	0.0527	0.368	1.467	4.77	15.13
3.8	0.0555	0.382	1.510	4.90	15.55
4.0	0.0582	0.396	1.551	5.03	15.96
4.2	0.0609	0.410	1.592	5.16	16.35
4.4	0.0636	0.423	1.631	5.28	16.73
4.6	0.0663	0.436	1.670	5.40	17.10
4.8	0.0690	0.449	1.707	5.51	17.48
5.0	0.0717	0.462	1.744	5.63	17.83
5.5	0.0782	0.492	1.833	5.91	18.70
6.0	0.0847	0.521	1.918	6.17	19.54
6.5	0.0911	0.548	1.999	6.42	20.34
7.0	0.0974	0.575	2.077	6.66	21.11
7.5	0.1036	0.600	2.152	6.90	21.86
8.0	0.1097	0.625	2.225	7.13	22.57
8.5	0.1157	0.649	2.295	7.35	23.27
9.0	0.1217	0.672	2.364	7.56	23.95
9.5	0.1276	0.694	2.430	7.77	24.60
9.9	0.1322	0.712	2.482	7.93	25.09

*Tabled value = peak frequency*spectral peak/variance = peak frequency/(2*effective width).

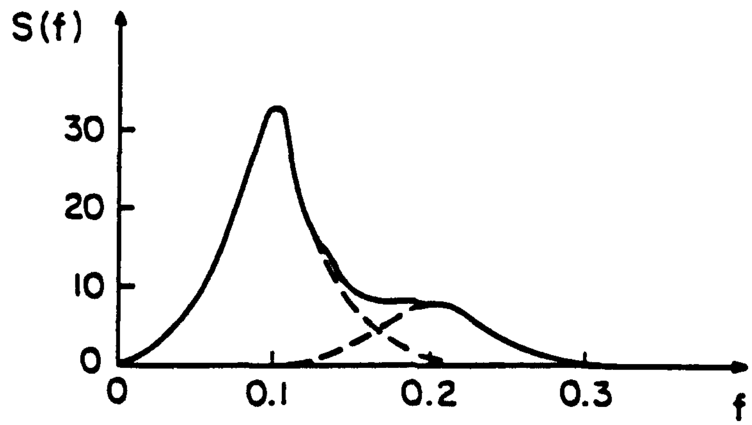

Figure 2
Decomposition of a bimodal spectral density.

As suggested by Ochi and Hubble (1976) the formula is particularly useful for the approximation of multimodal spectra. The formula also appears to have a much wider range of applicability than was investigated by the authors. Consider an example, shown in Figure 2, where the variance of the total spectrum is taken to be $\sigma^2 = 5\text{m}^2$. Suppose 2/3 of the variance is ascribed to the main mode. Then,

$$
\begin{aligned}
f_{01} &= 0.1\,\text{Hz}, \qquad S_{01} = 33.0, \qquad \sigma_1^2 = \frac{10}{3} \\
f_{02} &= 0.2\,\text{Hz}, \qquad S_{02} = 8.0, \qquad \sigma_2^2 = \frac{5}{3}
\end{aligned}
\tag{54}
$$

and

$$P_1 = 0.990, \qquad P_2 = 0.960 \tag{55}$$

For these values, the λ parameters are:

$$\lambda_1 = 1.748, \qquad \lambda_2 = 1.655 \tag{56}$$

The total spectrum is then modeled with a combination of these two modes.

In practice, modes are added to the model, the residuals examined, and further modes introduced until an adequate approximation is obtained. If an f^{-5} spectral slope is desired for high frequencies, a mode with $\lambda = 1$ can be added to the right-hand tail of the spectrum, while all the other modes at lower frequencies are kept much narrower.

Pierson-Moskowitz Spectrum (Pierson and Moskowitz, 1964). The Pierson-Moskowitz spectrum is used extensively by ocean engineers as one of the most representative for waters all over the world (Sarpkaya and Isaacson, 1981; Chakrabarti, 1987):

$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} e^{(-1.25(f/f_o)^{-4})} \qquad (57)$$

where: $\alpha = 8.1 \times 10^{-3}$ = Phillips' constant
f_o = peak frequency (Hz)
g = gravity (ft/s^2)

Bretschneider Spectrum (Bretschneider, 1959). The Bretscheider spectrum is designed to ensure the area m_o under the spectrum corresponds to $H_s/16$, which assumes a Rayleigh distribution of wave heights (Sarpkaya and Isaacson, 1981). The Bretscheider spectrum is based on the assumption that the spectrum is narrow-banded and the individual wave height and wave period follow the Rayleigh distribution (Chakrabarti, 1987):

$$S(f) = \frac{5 H_s^2}{16 f_o} \left(\frac{1}{(f/f_o)^5} \right) e^{[(-5/4)(f/f_o)^{-4}]} \qquad (58)$$

where: f_o = peak frequency
H_s = significant height

The relationship, $T_s = 0.946\ T_o$, where T_s = significant wave period, makes the Bretschneider and Pierson-Moskowitz (P-M) models equivalent (Chakrabarti, 1987).

JONSWAP Spectrum (Hasselman, et al., 1973). JONSWAP is a modification of the P-M spectrum to account for the effect of fetch restrictions and to provide for a much more sharply peaked spectrum (Sarpkaya and Isaacson, 1981; Barltrop and Adams, 1991):

$$S(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} e^{[(-5/4)(f/f_o)^{-4}]\gamma^a} \qquad (59)$$

where:

$$a = e^{[-(f-f_o)^2/2\sigma^2 f_o^2]}$$

$$\sigma = \begin{cases} \sigma_a = 0.07 \text{ for } f \le f_o \\ \sigma_b = 0.09 \text{ for } f > f_o \end{cases} \tag{60}$$

$$\alpha = 603.9\left[\left(\frac{H_s f_p^2}{g}\right)^{2.036}(1.0 - 0.298 \ln \gamma)\right]$$

$$\gamma = 3.3$$

Wallops Spectrum (Huang, et al., 1981).

$$S(f) = \frac{\beta g^2}{f^m f_o^{5-m}} e^{[(-m/4)(f_o/f)^4]} \tag{61}$$

where:

$$m = \left|\frac{\log(\sqrt{2}\,\pi\, e)^2}{\log 2}\right|$$

$$e = (\overline{\zeta^2})^{1/2}/\lambda_o$$

$$\beta = \frac{(2\pi e)^2 m^{(1/4)(m-1)}}{4^{(1/4)(m-5)}}\left\{\frac{1}{\Gamma[(1/4)(m-1)]}\right\} \tag{62}$$

$$(\overline{\zeta^2})^{1/2} = \frac{a}{\sqrt{2}}$$

λ_o = wavelength at spectrum peak

a = $H/2$

Γ = gamma function

Spectral Spreading Functions

Wrapped Normal Spreading Function. A convenient, easily interpreted, and mathematically tractable spreading formula is provided by the wrapped normal directional density (Mardia, 1972),

$$D(\theta;f) = \frac{1}{2\pi} + \sum_{n=1}^{\infty} \frac{e^{-n^2c^2/2}}{\pi} \cos[n(\theta - \theta_0)]$$
$$= \sum_{q=-\infty}^{\infty} \frac{\exp\{-(\theta - \theta_0 - 2\pi q)^2/2c^2\}}{\sqrt{2\pi}\, c} \tag{63}$$

where c is the circular standard deviation in radians. The first formula is best if $c > \pi$, while the second is better if $c < \pi$.

For the usual case, where $c << \pi/2$, the half-peak width of the spreading function is:

$$\theta_{HP} = c\sqrt{8 \log 2} \tag{64}$$

Von Mises Function (Mardia, 1972). Other common models from directional statistics are the von Mises formula (here I_0 = modified Bessel function of order zero):

$$D(\theta;f) = \frac{\exp[a\cos(\theta - \theta_0)]}{2\pi I_0(a)} \tag{65}$$

with half-peak width of

$$\theta_{HP} = 2\cos^{-1}\left(1 - \frac{\log 2}{a}\right) \tag{66}$$

Generalized Cosine-Squared Function.

$$D(\theta;f) = K \cos^{2\alpha}\left(\frac{\theta - \theta_0}{2}\right) \tag{67}$$

with half-peak width

$$\theta_{HP} = 4\cos^{-1}[(0.5)^{1/2\alpha}] \tag{68}$$

A reasonable set of equivalent values for c, a, and α are obtained by equating the half-peak widths,

$$c\sqrt{8\log 2} = 2\cos^{-1}\left(1 - \frac{\log 2}{a}\right) = 4\cos^{-1}\left[(0.5)^{1/2\alpha}\right] \tag{69}$$

and then solving for one in terms of the other.

Water Particle Kinematic Stretching

Wave properties above mean water level are not really defined within linear wave theory. Yet, this region is of profound importance to the design of engineering structures. Various approximate schemes to remedy this difficultly, at least approximately, have been suggested (Rodenbusch and Forristall, 1986).

Functional Extrapolation. The most simple procedure would be to use the formulas directly from linear wave theory, with z given values above mean water level. If z is measured positively downward from mean water lever, this would involve inserting negative z values into the formulas for the kinematics. The derivation for linear wave theory, however, assumes that $0 < z < d$. As a consequence, this approach leads to predictions of velocities at the crest that are much too large.

Truncation. Another simple procedure is to use, for the wave kinematics above mean water level, the value of those same kinematics at $z = 0$. However, that doesn't seem to lead to reasonable values either.

Linear Extrapolation. Linear extrapolation is based on using the rate of change of the wave property with respect to z, at $z = 0$ and linearly extrapolating for values above mean water level.

Let

$$\begin{aligned} \rho_0 &= \text{wave property value at } z = 0 \\ \rho &= d\rho(z)/dz \text{ at } z = 0 \end{aligned} \tag{70}$$

Then the linear extrapolation formula is:

$$\rho(z) = \rho_0 + \rho_0' z \tag{71}$$

Reid-Wheeler Stretching. Another scheme that has had some use in published engineering studies (Wheeler, 1969) consists of proportionally stretching the wave property at $z = 0$ up to $z = -\eta$, where η is the water level elevation above mean water level (i.e., positive upward). Then, the linear wave theory is computed with the vertical coordinate equal to z_s, where:

$$(d - z_s)/d = (d - z)/(d + \eta) \tag{72}$$

Because $-\eta < z < d$, it follows that

$$z_s/d = 1 - (d - z)/(d + \eta) \tag{73}$$

is always between 0 and 1.

Delta Stretching. Studies by Rodenbusch and Forristall (1986) suggest the following empirical procedure, which seems to produce fair agreement with field test measurements. Let $0 \leq \Delta \leq 1.0$ and $0 \leq d_\Delta \leq d$ be calibration numbers and define z_Δ so that,

$$(d_\Delta - z_\Delta)/(\eta\,\Delta + d_\Delta) = (d_\Delta - z)/(d_\Delta + \eta) \tag{74}$$

If $\Delta = 0$ and $d_\Delta = d$, then the Reid-Wheeler stretching listed above is produced. If $\Delta = 1$ and $d_\Delta = d$, then $z_\Delta = z$ and the use of z_Δ would correspond to functional extrapolation. Rodenbusch and Forristall suggest the following empirical rules:

1. If $d_\Delta \leq z \leq d$, use z directly in the linear wave theory.

2. If $z < d_\Delta$, compute z_Δ: (a) if $0 \leq z_\Delta \leq d_\Delta$, use z_Δ to compute the wave kinematics: (b) if $z_\Delta < 0$, compute the wave property by the linear extrapolation as:

$$\rho(z) = \rho_0 + \rho_0' z_\Delta \tag{75}$$

After studying various field data, Rodenbusch and Forristall found fairly satisfactory values for Δ and d_Δ to be:

$$\Delta = 0.3, \qquad d_\Delta = 2\,\sigma_\eta \tag{76}$$

where σ_η is the standard deviation of the sea surface.

Gamma Extrapolation. Another formula, suggested here as purely conjectural, may be based on the gamma function. Suppose the kinematic property above mean water level is modeled by:

$$\rho_s(z) = c\, e^{-(z+\eta)/\beta} (z+\eta)^{\alpha-1} \tag{77}$$

for $-\eta \le z \le 0$. Let ρ_0 and ρ_0' be as defined for linear extrapolation. If the gamma curve is matched to have $\rho_s(z) = \rho$ and $\rho_s' = \rho_0'$, and if the mode of the gamma is forced to occur at $z = \gamma\eta$, where $0 < \gamma \le 1$, it is easy to define α, β, and $\rho(z)$:

$$\beta = \frac{(\gamma - 1)\,\rho_0}{\rho_0'} \tag{78}$$

$$\rho(z) = \rho_0 \left(\frac{z+\eta}{\eta}\right)^{\alpha-1} \exp(-z/\beta) \tag{79}$$

$$\alpha = 1 + \frac{\eta\,\gamma}{\beta} \tag{80}$$

This curve has several appealing features in that it can be forced to attain the maximum just below the water surface, but is zero at the air-water interface (where it presumably would drop abruptly to zero). Also, it matches the kinematics at $z = 0$ and has the matching slope there. However, it has not been matched against real data, so it is suggested here as only an interesting possibility.

Comparisons with Data. Comparisons of the delta stretch procedure for three different data sets, one from a wave tank study and two from field measurements, are reported by Rodenbusch and Forristall (1986). They showed fairly good empirical correspondence, although there were discrepancies. Stretching is, at best, an empirical ad hoc procedure and should not be expected to yield perfect prediction.

RANDOM SIMULATION OF IRREGULAR WAVES

The most common assumption used for random simulations of irregular waves is that the wave properties form a Gaussian stochastic process. This means that all the wave properties at any selected set of times and locations follow a multivariate normal probability law. It is also usually assumed that separate components are independent of each other and that the real and imaginary parts of the complex amplitudes are independent of each other. As shown in the

Appendix, the real and imaginary parts of the amplitude, A_{mj}, are independent and normally distributed if, and only if, the ϕ_{mj} are uniform on (0, 2π), the a_{mj} are Rayleigh distributed, and ϕ_{mj} and a_{mj} are independent of each other.

The Rayleigh random variable here has a distribution function:

$$F(a_{mj}) = 1 - \exp\left(-a_{mj}^2/2\,\sigma_{mj}^2\right) \tag{81}$$

where

$$\sigma_{mj}^2 = S(f_m)\,D(\theta_j;f_m)\,\Delta f\,\Delta\theta/2 \tag{82}$$

A convenient way to generate a random number with a specified probability is to produce a uniform random number on the interval (0,1) and then to equate this to the distribution function for the desired special random number (Zelen and Severo, 1964, p. 950). Let U_1 be a uniform random number. Then,

$$F(a_{mj}) = U \tag{83}$$

gives

$$a_{mj} = \sigma_{mj}\sqrt{-2\log(1-U_1)} \tag{84}$$

These can be combined to obtain a random simulation of:

$$A_{mj} = a_{mj}e^{-i\phi_{mj}} \tag{85}$$

An alternative and equally good approach is to directly produce two independent standard normal random numbers, Z_1 and Z_2. Then,

$$A_{mj} = \sigma_{mj}(Z_1 + i\,Z_2) \tag{86}$$

Either method can be used to build up a simulation of the complex amplitude matrix, from which all the linear wave properties can be produced through the application of the appropriate transfer functions.

It is often convenient to skip the Rayleigh part of the simulation, set $a_{mj} = \sigma_{mj}$, and use only the random phase part. If the directional spreading is not too narrow, so that at least four or five energetic directional bands are added at each frequency, a central limit theorem behavior makes the resulting wave properties behave as multivariate normal random variables with the correct spectra and cross spectra; this is called the random phases model (Borgman, 1982b, p. 412). However, *if the Rayleigh part is retained, all components will exactly introduce Gaussian* behavior. An example of wave profile simulation is given in Figure 3.

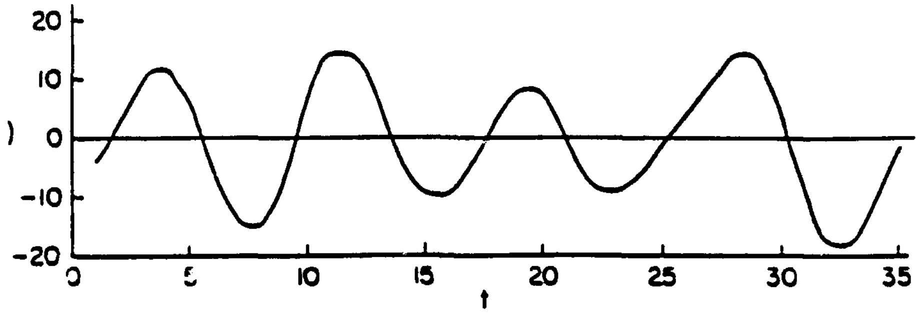

Figure 3
An unconditional simulation of a wave profile (variance = 50, λ = 1.0, f_0 = 0.1, principal direction = 60° (from), spreading standard deviation = 20°).

The user of multidirectional random phase wave simulations or the Rayleigh amplitude wave simulations should carefully distinguish between the input spectral density (i.e., the theoretical spectral density of the population) and the sample spectral density that can be computed from the simulations. The sample spectral density will contain ordinary sampling variations from the input spectra, and, thus, may depart rather substantially from the input function, both in overall variance and even in functional shape. If the sample spectra are computed by FFT procedures, the deviations between the sample and input spectral densities will be related to a chi-squared probability density.

Occasionally, users choose to simulate with the random phases model for unidirectional wave trains because this has the advantage that every simulation has its sample spectral density exactly equal to the input spectral density. Comparisons of platform response to wave forces for such simulations then show the variations possible between seas that exactly have the specified spectral density. These simulations are examples of what may be called constrained simulations. That is, the simulation is forced to have exactly some particular behavior (specified spectrum in the unidirectional random phases simulation case) that it would not have in a freely varying simulation.

The unidirectional random phases simulation has the additional property that it is approximately Gaussian in the time domain (unless the spectrum is very narrow-banded) through a central limit theorem. However, it is not Gaussian in the frequency domain, because the FFT coefficients are of the form $A_m = c \exp(-i\Phi)$, where c is a deterministic constant and Φ is a uniform random variable on $(0, 2\pi)$.

A multidirectional random phases simulation is approximately Gaussian, even in frequency, because now $A_m = \Sigma c_i \exp(i\Phi_j)$ and the summing over directions is enough to cause approximate normality, at least in the nonextreme ranges of simulated data, through a central limit theorem. A central limit theorem formulation suitable to these derivations is given by Takano (1954) in a doubly subscripted, multivariate format. A typical derivation of the Gaussian properties of random phase models is provided by Brown (1967).

Another property of simulations that should be understood by users is that of ergodicity. In words, a process that is ergodic should have its probabilistic behavior for infinitely long time intervals be the same as the probability laws for the ensemble of possibilities at a single time or at several specified times. Because the simulation procedures presented previously are based on a fixed matrix of complex amplitudes, A_{mj}, determined once and for all by a finite set of random numbers, the time series given by the A_{mj} will always have the random bias introduced by the random number sampling variability, regardless of how long the time series is made. Thus, the simulation process provided is not ergodic as N tends to infinity (see Jefferys, 1987; Tucker, et al., 1984).

Tucker is particularly concerned that the standard deviation of the wave property be the same from (x, y) location to (x, y) location, because such behavior is important in his applications. In real wave data, if the time interval of sampling is long enough, and if the wave field statistics remain stationary over the multiple hours of record required, ergodicity will force the variance at every spatial location to be the same. However, in the usual 20 minutes of most wave data, there will be substantial differences in variance from location to location. Real world data show the same spatially varying sampling variability as the simulations.

Simulations of directional seas on the multidirectional case can be constrained to have exactly a specified frequency spectral density. The procedure is as follows. The matrix of complex amplitudes A_{mj} is simulated with full sampling variability as shown in Equation 86. Then the sequence $\{A_{mj};\ j = 1,\ldots,J\}$ is summed as:

$$\beta = \frac{\sum_{j=1}^{J} |A_{mj}|^2}{\sum_{j=1}^{J} \sigma_{mj}^2} \tag{87}$$

Finally, the original A_{mj} are divided by $B^{1/2}$ to get a new set of A_{mj} at that frequency. This is repeated at the other frequencies so that the simulation will have exactly the specified $S(m\Delta f)$ spectral density.

A similar procedure can be used to constrain the directional simulation to have a directional spectral density that exactly agrees with the theoretical population directional spectral density. Here the complex amplitude matrix is simulated as before with complete sampling variation freedom. Then the $A_{mj} = U_{mj} - iV_{mj}$ are replaced by:

$$A_{mj}\,\sigma_{mj}/(U_{mj}^2 + V_{mj}^2)^{1/2} \tag{88}$$

The important thing to remember is that a simulation is always artificial. It will retain some properties of the real world but will violate others. There is not one correct way to compute simulations, but various ways that are appropriate to particular applications. Jefferys (1987) and Tucker, et al. (1984) provide a real service in emphasizing the dangers in blindly using a simulation procedure without appreciating the artificial aspects of the simulation. However, one should not go too far the other way and condemn a simulation procedure for all applications, when it is perfectly satisfactory in many problems where those particular defects are unimportant.

Usually, ergodicity is not particularly important for studies of wave forces on offshore structures as long as the sampling variability for a given finite interval of simulation is appreciated. A typical treatment involves looking at the force behavior in a number of separate simulations. This represents a sample "ensemble set" and convergence to the correct probability laws or correct spectral density will occur by averaging over an increasing number of simulation sets.

A very interesting application of the unconditional simulation procedures given in the preceding pages has been developed by M. J. Briggs at the Coastal Engineering Research Center, Vicksburg, Mississippi (Briggs, et al., 1987). He simulates water level elevation time histories at directional paddle locations with an input directional spectral density, then converts these time series to stroke magnitude, and finally produces a directional wave field within the wave basin. He then reverses the procedure by using an array of model tank wave staffs in the center of the basin as input to a directional spectrum analysis program and estimates the directional spectral density that has been developed. Interestingly enough, the estimated spectra agree quite well with the original input spectra, within the usual range of sampling and estimate variability.

CONDITIONAL SIMULATION OF IRREGULAR WAVES

The various linear properties of the irregular wave train will be assumed to follow the multivariate normal probability law over space and time. Within this framework, conditional probability laws are themselves multivariate normal and an elaborate theory can be constructed. It will be assumed in the following that the reader has some familiarity with multivariate statistical analysis and matrix theory.

The problem that will be considered here is the following. Suppose wave properties have been measured for some set of wave conditions of interest, and the directional spectrum $S(f,\theta)$ has also been estimated or an acceptable model has been found. What are reasonable time series for other wave properties co-occurring with the measured set, or for the same wave properties at times other than the measurement intervals?

The wave properties that were not measured are stochastic processes that are to some degree correlated with measured data. Depending on the extent and amount of the actual data, the nonmeasured wave properties may be constrained to strong agreement with the measurements or may only be weakly related to them.

Conditional simulation of wave property time series statistically consistent with a specified measurement set provides a very powerful approach to certain ocean engineering problems. The method does not provide a single determination of the nonmeasured time series, but rather one for each simulation. The range of variation from simulation to simulation provides a measure of how strongly the measurements determine the structure behavior.

Although these methods are quite new in ocean engineering, they have had some use in the petroleum industry (Rodenbusch and Forristall, 1986) and the author is aware of at least two other studies where these methods were highly successful in applications to ocean waves (Vartdal, Krogstad, and Barstow, 1989). Rather similar methods in geostatistics and mining (Journel, 1974; Borgman, Taheri, and Hagan, 1983) have been used for spatial random fields to some extent for several decades.

The usual computer simulation of waves (Borgman, 1982b) satisfying a specified model for the directional spectral density suffers from a serious practical defect if one is primarily interested in producing very large waves. Most simulations produce average waves. Very long computer runs are required to capture the occurrence of an extra large wave. However, the conditional simulation starts with the inclusion of a large wave profile embedded into the wave train. The various associated kinematics are produced consistent with this large wave, for each simulation. Every computer run produces another simulation that is appropriate to the engineering analysis.

A large variety of problems can be approached from the conditional simulation perspective. In addition to using measured storm wave intervals to condition on, one could introduce wave groups of three or four large waves in sequences, condition on these to produce the associated kinematics, and then impose all this onto a structure for dynamic behavior. Another type of investigation might be to study the variability in kinematics associated with a large Stokes'-type wave profile, when a directional spectrum is present with various degrees of spreading. Undoubtedly many more diverse applications will be found as these methods become more widely known.

Multivariate Normal Probability Law

The n component vector **V** with mean μ and covariance matrix C, will be said to behave according to a multivariate normal probability law if its probabilities of occurrence follow the probability density:

$$F_V(v) = \left[(2\pi)^{n/2}\sqrt{|C|}\,\right]^{-1} \exp\left[-(v-\mu)^T C^{-1}(v-\mu)/2\right] \tag{89}$$

where the superscript T denotes the transpose, $|C|$ is the determinant of C, and C^{-1} is the inverse of C. The inverse of C in the exponent can be replaced with a generalized inverse, C^+, if suitable restrictions to the domain space are introduced (Pringle and Rayner, 1971, pp. 70-72). It is often more convenient to use a shorthand notation for the specification of multivariate normality as:

$$\mathbf{V}: N(\mu, C) \tag{90}$$

This is read as **V** is multivariate normal with mean μ and covariance matrix C.

Conditional Normal Density

Suppose the random vector **V** can be broken up, or partitioned, into two parts, $\mathbf{V}_1$ and $\mathbf{V}_2$, and that the combined vector satisfies:

$$\mathbf{V} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} : N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}\right) \tag{91}$$

That is,

$$E[\mathbf{V}_1] = \mu_1 \tag{92}$$

$$E[\mathbf{V}_2] = \mu_2 \tag{93}$$

$$\text{covariance matrix of } (\mathbf{V}_1) = E\left[(\mathbf{V}_1 - \mu_1)(\mathbf{V}_1 - \mu_1)^T\right] = \mathbf{C}_{11} \tag{94}$$

$$\text{covariance matrix of } (\mathbf{V}_2) = \mathbf{C}_{22} \tag{95}$$

$$\text{cross-covariance matrix of } (\mathbf{V}_1, \mathbf{V}_2) = E\left[(\mathbf{V}_1 - \mu_1)(\mathbf{V}_2 - \mu_2)^T\right] = \mathbf{C}_{12} \tag{96}$$

Suppose specific values for $\mathbf{V}_1$ are assigned numerically as $\mathbf{V}_1 = \mathbf{v}_1$. Here $\mathbf{v}_1$ is a list of numbers, while $\mathbf{V}_1$ is the abstract random vector. Then the probability density for $\mathbf{V}_2$, given $\mathbf{V}_1 = \mathbf{v}_1$, is (Anderson, 1958):

$$(\mathbf{V}_2 | \mathbf{V}_1 = \mathbf{v}_1):\ N\left(\mu_2 + \mathbf{C}_{12}^T \mathbf{C}_{11}^{-1}(\mathbf{v}_1 - \mu_1),\ \mathbf{C}_{22} - \mathbf{C}_{12}^T \mathbf{C}_{11}^{-1} \mathbf{C}_{12}\right) \tag{97}$$

In its simplest form, conditional simulation is just any procedure for producing a vector $\mathbf{V}_2$ with the mean vector and covariance matrix given above.

Conditional Simulation Method

A straightforward method (Borgman, Taheri, and Hagan, 1983) is given by a two-step procedure:

1. Unconditionally simulate $\mathbf{V}$, without reference to $\mathbf{V}_1 = \mathbf{v}_1$. Let $\mathbf{V}_1$ and $\mathbf{V}_2$ denote these unconditional simulations.

2. A conditional simulation is provided by:

$$(\mathbf{V}_2 | \mathbf{V}_1 = \mathbf{v}_1) = \mathbf{C}_{12}^T \mathbf{C}_{11}^{-1}(\mathbf{v}_1 - \mathbf{V}_1) + \mathbf{V}_2 \tag{98}$$

All of the techniques based on this procedure require substantial facility in deriving and computing the covariances in $\mathbf{C}_{11}$ and $\mathbf{C}_{12}$ and in inverting $\mathbf{C}_{11}$. For example, consider two general wave properties of the form of Equation 25 combined with Equation 39. Let $\lambda = 1,2$ [with Δt = time increment, $\Delta f = 1/(N\Delta t)$]:

$$\begin{aligned} B_\lambda(m\,\Delta f; x,y,z) &= \sum_{j=1}^{J} A_{mj}\, G_\lambda(m\,\Delta f, z)\, T_\lambda(m\,\Delta f)\, H_\lambda(j\,\Delta\theta) \\ &\quad * \exp\left[-i\,\beta_0 k_m\{x\cos(j\,\Delta\theta - \theta_x) + y\cos(j\,\Delta\theta - \theta_y)\}\right] \end{aligned} \tag{99}$$

for $0 < m < N/2$. Set

$$B_\lambda(0: x,y,z) = 0$$

$$B_\lambda(N\,\Delta f/2;x,y,z) = 0 \qquad (100)$$

$$B_\lambda([N-m]\,\Delta f;x,y,z) = B_\lambda^*(m\,\Delta f;x,y,z)$$

for $N/2 < m < N$. The wave properties, in time domain, are given by:

$$b_\lambda(n\,\Delta t;x,y,z) = \sum_{m=0}^{N-1} B_\lambda(m\,\Delta f;x,y,z)\exp(-i\,2\,\pi\,m\,n/N) \qquad (101)$$

for $n = 0,1,2,\ldots,N\text{-}1$.

The covariances between two time domain wave properties are developed in Theorem D of the Appendix. That is:

$$Q_{mj}^{(\lambda)} = G_\lambda(m\,\Delta f,z)\,T_\lambda(m\,\Delta f)\,H_\lambda(j\,\Delta\theta)$$

$$* \exp\left[-i\,\beta_0\,k_m\{x\cos(j\,\Delta\theta-\theta_x)+y\cos(j\,\Delta\theta-\theta_y)\}\right] \qquad (102)$$

Equation 2 of the theorem gives the covariance between the two wave properties one at time $n\,\Delta t$ and the other at time $n'\,\Delta t$. The location coordinates (x_1,y_1,z_1) for the first wave property must be substituted into $Q_{mj}^{(1)}$, while the corresponding values for location of the second wave property (x_2,y_2,z_2) must be substituted in $Q_{mj}^{(2)}$. Stated in terms of integral,

$$E\left[b^{(1)}(t)\,b^{(2)}(t+\tau)\right] = \int_{-\infty}^{\infty}\int_0^{2\pi} S(f,\theta)\,Q^{(1)*}(f,\theta)\,Q^{(2)}(f,\theta)\exp(i\,2\,\pi\,f\,\tau)\,d\theta\,df \qquad (103)$$

The computation of the wave property covariances can be quite a task. Some simplification and acceleration of the calculations can be achieved by introducing the Fast Fourier Transform algorithm. However, this still leaves a substantial effort in manipulating C_{12} and computing C_{11}^{-1} for the often large matrices that occur in realistic applications.

Conditional Simulation of Complex Wave Amplitudes

Great savings in computer run time, as well as the achievement of very substantial simplification and clarification of the model in application, is obtained by centering attention on the matrix of complex wave amplitude, $A_{mj} = U_{mj} - iV_{mj}$ introduced previously. The A_{mj} are conditionally simulated, given the specified wave properties and the selected directional spectral density.

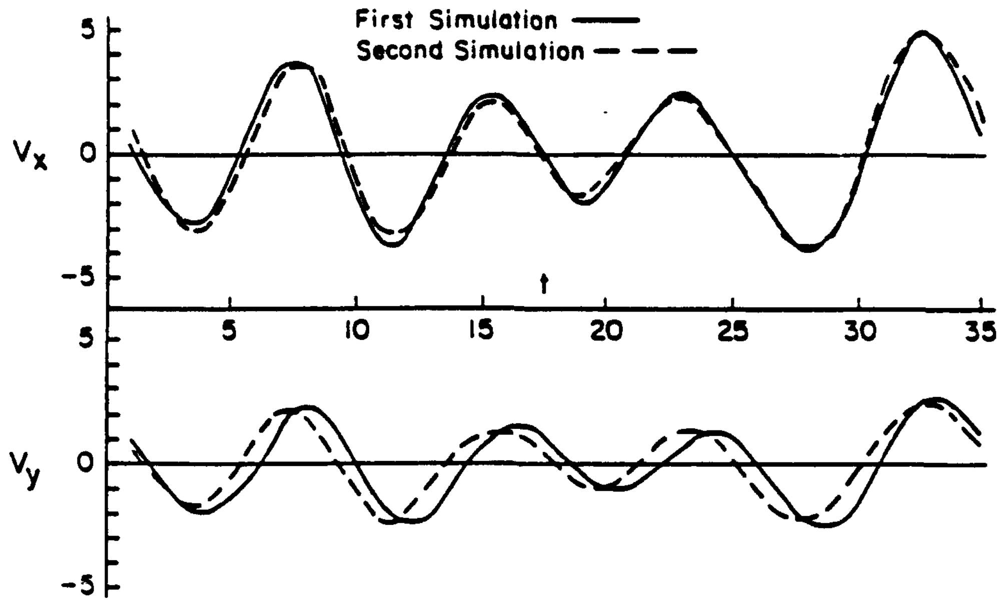

Figure 4
Two conditional simulations of water particle velocities, given the profile in Figure 3. (Waves from heading of 60°: x axis east: y axis north: velocities at 15.7 m below mean water level.)

The actual calculation of these complex amplitude simulations requires a fairly sophisticated introduction of frequency domain methods as interrelated to time domain data. The Appendix provides a very succinct summary of a fundamental set of theorems giving a theoretical framework for the discrete Fourier transform.

Frequency Domain Conditioning

A common special case in applications arises when the wave profile or other wave properties have been measured over the full time interval of interest. Perhaps this represents an interval of time during a historical storm when wave action was quite high. A simulation of the wave kinematics throughout the wave, conditioned to agree with the occurrence of the measured time series, is needed for wave force analysis. What is the most efficient way to develop this conditional simulation?

A very effective technique is to first perform FFT on the measured time series to obtain the Fourier coefficients, and then to conditionally simulate each $A_{mj} = U_{mj} - iV_{mj}$ separately, given the Fourier coefficients $\{B_\lambda(m\ \Delta t;\ x, y, z),\ \lambda = 1, \ldots,$ number of measured time series$\}$ for the measured wave. This will be illustrated with the case for one measured time series and the case for two measured time series. Figure 4 gives examples of such simulations.

Consider first the situation for one measured time series $\{b(n\,\lambda t), n = 0,1,\ldots,N\text{-}1\}$, with FFT coefficients:

$$B(m\,\Delta f) = \sum_{m=0}^{N-1} b(n\,\Delta t)\exp(-i\,2\,\pi\,m\,n/N) \tag{104}$$

with the Q_{mj} from Equation 102, $B(m\Delta f)$ can be related to the complex-valued wave amplitude as:

$$B(m\,\Delta f) = \sum_{j=1}^{J} A_{mj}\,Q_{mj} \tag{105}$$

The goal, then, is to conditionally simulate U_{mj} and V_{mj}, give $B(m\,\Delta f)$.

The following structural assumptions will be made:

1. All time series will be assumed to be covariance-stationary with zero mean.

2. All time series will be repeated periodically.

3. $A_{0j} = A_{N/2,j} = 0$ for all $j = 1,2,\ldots,J$.

The imposition of the second assumption has the effect of making the last part of the simulation have a correlation with the first part. If this causes improper behavior in the application of the series, about three wave periods of simulation should be deleted at the end of the simulated time series, before it is used. This just means that N is made a little larger than is needed in the application to adjust for the deletion of some time steps at the end of the simulation. The addition of the periodicity assumption produces such a clean, elegant statistical structure in the Fourier coefficients that it is definitely worth the trouble of compensating for the minor distortions introduced by the assumption. The imposition of the third assumption just forces the simulations to have mean zero and to have no energy at the Nyquist frequency.

The covariance matrix for A_{mj} and

$$B(m\,\Delta f) = \Phi_m + i\,\Psi_m \tag{106}$$

can be constructed from Theorem D in the Appendix. It is best stated in terms of the real random variables U_{mj}, V_{mj}, Φ_m and Ψ_m. For $0 < m < N/2$:

$$\mathrm{Cov}\begin{bmatrix}\Phi_m\\ \Psi_m\\ U_{mj}\\ V_{mj}\end{bmatrix} = \left(\frac{\Delta f\,\Delta\theta}{2}\right)$$

$$* \begin{bmatrix} \sum_{j=1}^{J} S_{mj}|Q_{mj}|^2 & 0 & S_{mj}\mathrm{Re}(Q_{mj}) & S_{mj}\mathrm{Im}(Q_{mj}) \\ 0 & \sum S_{mj}|Q_{mj}|^2 & S_{mj}\mathrm{Im}(Q_{mj}) & -S_{mj}\mathrm{Re}(Q_{mj}) \\ S_{mj}\mathrm{Re}[Q_{mj}] & S_{mj}\mathrm{Im}(Q_{mj}) & S_{mj} & 0 \\ S_{mj}\mathrm{Im}(Q_{mj}) & -S_{mj}\mathrm{Re}[Q_{mj}] & 0 & S_{mj} \end{bmatrix} \tag{107}$$

The actual conditional simulations are then just an application of Equation 98, with:

$$C_{11} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \sum_{j=1}^{J} S_{mj}|Q_{mj}|^2\,\Delta f\,\Delta\theta/2 \tag{108}$$

$$C_{12} = \begin{bmatrix}\mathrm{Re}(Q_{mj}) & \mathrm{Im}(Q_{mj})\\ \mathrm{Im}(Q_{mj}) & -\mathrm{Re}(Q_{mj})\end{bmatrix} \frac{S_{mj}\,\Delta f\,\Delta\theta}{2} \tag{109}$$

$$V_1 = \begin{bmatrix}\Phi_m\\ \Psi_m\end{bmatrix}, \text{ from the measured time series} \tag{110}$$

The unconditional simulation of:

$$V_2 = \begin{pmatrix}U_{mj,us}\\ V_{mj,us}\end{pmatrix} \tag{111}$$

can be produced easily from two independent standard normal random numbers (Z_1, Z_2) with the formulas:

$$U_{mj,us} = Z_1(S_{mj}\,\Delta f\,\Delta\theta/2)^{1/2} \tag{112}$$

$$V_{mj,us} = Z_2 (S_{mj} \Delta f \, \Delta\theta/2)^{1/2} \tag{113}$$

This is repeated for the other j-indices with additional pairs of j-indices with additional pairs of independent standard normal random numbers. The unconditional simulations of Φ_m and Ψ_m are completely determined by the set of values $\{(U_{mj,us}, V_{mj,us}),\ j = 1,2,\ldots,J\}$ as:

$$\Phi_{m,us} + i\,\Psi_{m,us} = \sum_{j=1}^{J} (U_{mj,us} - i\,V_{mj,us})\, Q_{mj} \tag{114}$$

Equation 98 for the conditional simulation becomes:

$$(V_2 | V_1 = v_1) = \begin{pmatrix} U_{mj,cs} \\ V_{mj,cs} \end{pmatrix}$$

$$= \begin{bmatrix} Re(Q_{mj}) & Im(Q_{mj}) \\ Im(Q_{mj}) & -Re(Q_{mj}) \end{bmatrix} \frac{S_{mj}\, \Delta f\, \Delta\theta/2}{\sum_{j=1}^{J} S_{mj} |Q_{mj}|^2 \Delta f\, \Delta\theta/2}$$

$$* \left[\begin{pmatrix} \Phi_m \\ \Psi_m \end{pmatrix} - \begin{pmatrix} \Phi_{m,us} \\ \Psi_{m,us} \end{pmatrix} \right] + \begin{pmatrix} U_{mj} \\ V_{mj} \end{pmatrix} \tag{115}$$

The validity of Equation 115 can be verified by confirming that:

$$B_{m,cs} = \sum_{j=1}^{J} (U_{mj,cs} - i\,V_{mj,cs})\, Q_{mj} \tag{116}$$

is the same as B(m Δf) in Equation 106. That is, the complex amplitudes produced by conditional simulation do, in fact, produce the FFT coefficients for the conditioning time series. The proof of Equation 116 follows from:

$$(U_{mj,cs} - i\,V_{mj,cs}) = (1,-i)\begin{pmatrix} U_{mj,cs} \\ V_{mj,cs} \end{pmatrix} \tag{117}$$

Applying this to Equation 115, one gets, after some algebra,

$$(U_{mj,cs} - i\,V_{mj,cs}) = \frac{Q^{*}_{mj}\,S_{mj}\,\Delta f\,\Delta\theta}{\sum_{j=1}^{J} S_{mj}\left|Q_{mj}\right|^2 \Delta f\,\Delta\theta}\left[B_m - B_{m,us}\right] + A_{mj,us} \tag{118}$$

The substitution of Equation 118 into Equation 116 gives:

$$B_{m,cs} = B_m - B_{m,us} + \sum_{j=1}^{J} A_{mj,us} Q_{mj} = B_m \tag{119}$$

which is the verifying equality.

The covariance is more complicated if there are two time series that are simultaneously measured, which are to be used for conditioning. Then,

$$B^{(1)}_m = \sum_{j=1}^{J} A_{mj} Q^{(1)}_{mj} = \Phi^{(1)}_m + i\,\Psi^{(1)}_m \tag{120}$$

$$B^{(2)}_m = \sum_{j=1}^{J} A_{mj} Q^{(2)}_{mj} = \Phi^{(2)}_m + i\,\Psi^{(2)}_m \tag{121}$$

are the conditioning FFT coefficients. The covariance matrix which must be evaluated is that for the six variables $(\Phi^{(2)}_m, \Psi^{(2)}_m, \Phi^{(2)}_m, \Psi^{(2)}_m, U_{mj}, V_{mj})$. For this,

$$C_{11} = \sum_{j=1}^{J} S_{mj} \begin{bmatrix} |Q^{(1)}_{mj}|^2 & 0 & \mathrm{Re}\left[Q^{(1)*}_{mj}\, Q^{(2)}_{mj}\right] & \mathrm{Im}\left[Q^{(1)*}_{mj}\, Q^{(2)}_{mj}\right] \\ 0 & |Q^{(1)}_{mj}|^2 & -\mathrm{Im}\left[Q^{(1)*}_{mj}\, Q^{(2)}_{mj}\right] & -\mathrm{Re}\left[Q^{(1)*}_{mj}\, Q^{(2)}_{mj}\right] \\ \mathrm{Re}\left[Q^{(1)*}_{mj}\, Q^{(2)}_{mj}\right] & -\mathrm{Im}\left[Q_{m j_{(1)*}}\, Q^{(2)}_{mj}\right] & |Q^{(2)}_{mj}|^2 & 0 \\ \mathrm{Im}\left[Q^{(1)*}_{mj}\, Q^{(2)}_{mj}\right] & \mathrm{Re}\left[Q^{(1)*}_{mj}\, Q^{(2)}_{mj}\right] & 0 & |Q^{(2)}_{mj}|^2 \end{bmatrix} \frac{\Delta f\, \Delta\theta}{2} \qquad \text{....... (122)}$$

$$C_{12} = S_{mj} \begin{bmatrix} \mathrm{Re}\left[Q^{(1)}_{mj}\right] & \mathrm{Im}\left[Q^{(1)}_{mj}\right] \\ \mathrm{IM}\left[Q^{(1)}_{mj}\right] & -\mathrm{Re}\left[Q^{(1)}_{mj}\right] \\ \mathrm{Re}\left[Q^{(2)}_{mj}\right] & \mathrm{Im}\left[Q^{(2)}_{mj}\right] \\ \mathrm{Im}\left[Q^{(2)}_{mj}\right] & -\mathrm{Re}\left[Q^{(2)}_{mj}\right] \end{bmatrix} \frac{\Delta f\, \Delta\theta}{2} \qquad (123)$$

The conditional simulation proceeds in two steps as before. First, an unconditional simulation of U_{mj} and V_{mj} is formed with pairs of independent standard normal random numbers $\{Z_{mj1}, Z_{mj2};\ 0 < m < N/2,\ j = 1,2,\ldots,J\}$ with the formulas:

$$U_{mj,us} = Z_{mj1}\,(S_{mj}\ \Delta f\, \Delta\theta/2)^{1/2} \qquad (124)$$

$$V_{mj,us} = Z_{mj2}\,(S_{mj}\ \Delta f\, \Delta\theta/2)^{1/2} \qquad (125)$$

The unconditional simulation of $B^{(1)}_{m}$ and $B^{(2)}_{M}$ is achieved with Equation 105 as:

$$B^{(1)}_{m,us} = \sum_{j=1}^{J} (U_{mj,us} - i V_{mj,us})\, Q^{(1)}_{mj} \qquad (126)$$

$$B^{(2)}_{m,us} = \sum_{j=1}^{J} (U_{mj,us} - iV_{mj,us})\, Q^{(2)}_{mj} \tag{127}$$

The second step consists of substituting all these results into Equation 98. It can be shown, as before, that the conditional simulation of the A_{mj}, when used with the transfer functions $Q^{(1)}_{mj}$ and $Q^{(2)}_{mj}$, reproduce the FFT coefficients used in the conditioning. However, the details are too complicated for presentation here.

Time Domain Conditioning

Time domain conditioning is addressed in the following problem. Suppose one or more time series are measured over portions of the time interval of interest. The problem is to simulate conditionally the time series for the rest of the time interval and for other wave properties during the entire interval. Figure 5 provides an example of this technique.

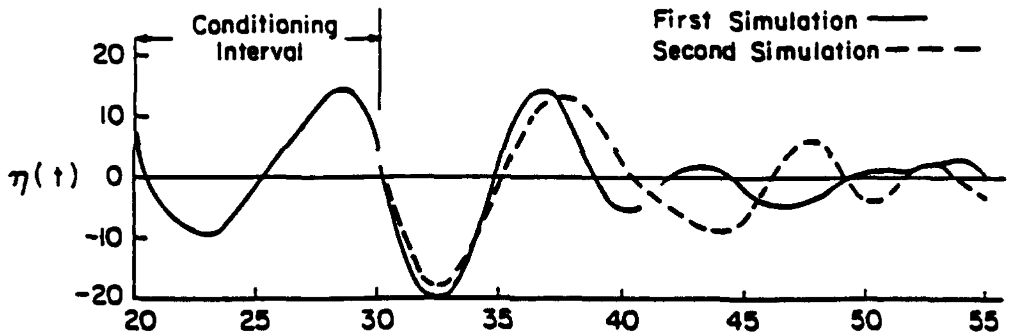

Figure 5
Two conditional simulations of the wave profile for 30 < t < 55, given the profile in Figure 3 for 0 < t < 30.

The technique will be illustrated with a single conditioning time series, consisting of the water level elevations, $\eta(n\ \Delta t)$. Suppose $\{\eta(n\ \Delta t),\ n = 0,1,2,\ldots,\nu\}$, where $\nu < N$ is known and is to be used for conditioning. The covariance matrix for $\{\eta(0),\eta(2\ \Delta t),\eta(2\Delta t),\ldots,\eta(N_0\ \Delta t),U_{mj},V_{mj}\}$ forms the basis for the simulation.

Let

$$c_k = \sum_{m=0}^{N-1} \sum_{j=1}^{J} S_{mj} \exp[i\, 2\, \pi\, k\, m/N]\, \Delta\theta\, \Delta f \tag{128}$$

Then C_{11} is:

$$C_{11} = \begin{bmatrix} c_0 & c_1 & c_2 & \cdots & c_\nu \\ c_1 & c_0 & c_1 & \cdots & c_{\nu-1} \\ c_2 & c_1 & c_0 & \cdots & c_{\nu-2} \\ \vdots & \vdots & \vdots & & \vdots \\ c_\nu & c_{\nu-1} & c_{\nu-2} & \cdots & c_0 \end{bmatrix} \tag{129}$$

and the C_{12} matrix is:

$$C_{12} = \begin{bmatrix} S_{mj}\, \Delta f\, \Delta\theta & 0 \\ S_{mj} \cos(2\, \pi\, m/N)\, \Delta f\, \Delta\theta & S_{mj} \sin(2\, \pi\, m/N)\, \Delta f\, \Delta\theta \\ S_{mj} \cos(4\, \pi\, m/N)\, \Delta f\, \Delta\theta & S_{mj} \sin(4\, \pi\, m/N)\, \Delta f\, \Delta\theta \\ S_{mj} \cos(6\, \pi\, m/N)\, \Delta f\, \Delta\theta & S_{mj} \sin(6\, \pi\, m/N)\, \Delta f\, \Delta\theta \\ \vdots & \vdots \\ S_{mj} \cos(2\, \pi\, m\nu/N)\, \Delta f\, \Delta\theta & S_{mj} \sin(2\, \pi\, m\nu/N)\, \Delta f\, \Delta\theta \end{bmatrix} \tag{130}$$

The conditional simulation proceeds by first using Equations 124 and 125 to get $U_{mj,us}$ and $V_{mj,us}$. Then the unconditional simulation of the $\eta(n\ \Delta t)$ is computed from:

$$\eta_{us}(n\, \Delta t) = \sum_{m=0}^{N-1} \sum_{j=1}^{J} A_{mj,us} \exp(i\, 2\, \pi\, mn/N) \tag{131}$$

Finally, Equation 98 is used to get the conditioned set of complex wave amplitudes with:

$$v_1 = \begin{bmatrix} \eta(0) \\ \eta(\Delta t) \\ \eta(2\,\Delta t) \\ \vdots \\ \eta(\nu\,\Delta t) \end{bmatrix}, \qquad V_1 = \begin{bmatrix} \eta_{us}(0) \\ \eta_{us}(\Delta t) \\ \eta_{us}(2\,\Delta t) \\ \vdots \\ \eta_{us}(\nu\,\Delta t) \end{bmatrix} \tag{132}$$

The inversion of C_{11} can be done with special techniques for Toeplitz matrices (Golub and Van Loan, 1983, pp. 125-133) or with conjugate gradient iteration (Golub and Van Loan, 1983, pp. 362-369). Any technique that solves $C_{11}x = \mathbf{b}$ for x, given $\mathbf{b}$, can be used. If C_{11} is singular, the generalized inverse of C_{11} (Searle, 1983, pp. 212-226) can be used in place of C_{11}^{-1} in Equation 98.

TECHNIQUES FOR COMPLIANT STRUCTURES

The techniques for simulation presented previously are based on a fixed coordinate position (x, y, z). If the structure is moving, there are difficulties in following the structure from one position to another. One way to proceed would be to compute time series at each intersection for an extensive three-dimensional grid over the (x, y, z) space. Then the velocities and accelerations at any arbitrary space location could be interpolated. However, for a typical structure, this procedure requires such formidable amounts of computer storage as to be impossible on even large computers.

An alternative procedure is presented here that surmounts this difficulty and requires only moderate computer capacity. The procedure is based on the need for wave kinematics and other wave properties only within a relatively narrow rectangle of horizontal coordinate position as compared to the wavelengths involved. The approach involves the expansion of the wave properties in Legendre orthogonal polynomials in x, y, and z.

Let

$$\begin{aligned} p_n(x) &= b_0 + b_1 x + \dots + b_n x^n \\ p_n^*(x) &= b_0^* + b_1^* x + \dots + b_n^* x^* \end{aligned} \tag{133}$$

be the Legendre and shifted Legendre orthogonal polynomials of order n (Hochstrasser, 1964). The coefficients are selected so that:

$$\int_{-1}^{1} p_m(x)\, p_n(x)\, dx = \begin{cases} 0, & \text{if } m \neq n \\ \dfrac{2}{2n+1}, & \text{if } m = n \end{cases} \tag{134}$$

$$\int_{0}^{1} p_m^*(x)\, p_n^*(x)\, dx = \begin{cases} 0, & \text{if } m \neq n \\ \dfrac{1}{2n+1}, & \text{if } m = n \end{cases} \tag{135}$$

The Legendre polynomials are defined for $-1 < x < 1$ and the shifted Legendre polynomials are defined for $0 < x < 1$.

The first several Legendre polynomials are:

$$p_0(x) = 1 \tag{136}$$

$$p_1(x) = x \tag{137}$$

$$p_2(x) = (3x^2 - 1)/2 \tag{138}$$

$$p_3(x) = (5x^3 - 3x)/2 \tag{139}$$

$$p_4(x) = (35x^4 - 30x^2 + 3)/8 \tag{140}$$

$$p_5(x) = (63x^5 - 70x^3 + 15x)/8 \tag{141}$$

The shifted Legendre polynomials are related to the regular Legendre polynomials by the relation:

$$p_n^*(x) = p_n(2x - 1) \tag{142}$$

The use of orthogonal polynomials to approximate an arbitrary function g(x) defined on (-1,1) can be illustrated as follows. Suppose the approximation to be used is:

$$g(x) = \sum_{n=0}^{N} a_n p_n(x) \tag{143}$$

The coefficients a_n are chosen from a "least-squares" criterion. Let a_n be those values that minimize:

$$Q = \int_{-1}^{1} \left[g(x) - \sum_{n=0}^{N} a_n p_n(x) \right]^2 dx \tag{144}$$

Then,

$$\frac{\partial Q}{\partial a_k} = \int_{-1}^{1} 2 \left[g(x) - \sum_{n=0}^{N} a_n p_n(x) \right] p_k(x)\, dx \tag{145}$$

Q is at an extreme if $\partial Q/\partial a_k = 0$ for all $k = 0,1,2,\ldots N$. This reduces to:

$$\sum_{n=0}^{N} a_n \int_{-1}^{1} p_n(x)\, p_k(x)\, dx = \int_{-1}^{1} g(x)\, p_k(x)\, dx \tag{146}$$

But, by the orthogonality relation, this further reduces to:

$$a_k = \left(\frac{2k+1}{2} \right) \int_{-1}^{1} g(x)\, p_k(x)\, dx \tag{147}$$

The similar development for shifted Legendre polynomials gives:

$$a_k^* = (2k+1) \int_{0}^{1} g(x)\, p_k^*(x)\, dx \tag{148}$$

How can these relations be applied to ocean wave kinematics? The essential canonical form for a linear wave property is given in Equation 23. It should be noted that, in every case, G(z) is either 1.0 (water level elevation) or is of the form:

$$\frac{e^{-k_m z} + s_1 e^{-k_m(2d-z)}}{1 + s_2 e^{-2k_m d}} \tag{149}$$

where k_m is the wave number. Thus, the general wave property $b(n\,\Delta t)$ can be expressed from Equation 35 as:

$$b(n\,\Delta t) = \sum_{m=0}^{N-1} B_m e^{i2\pi mn/N} \tag{150}$$

where B_m is the FFT coefficient given by:

$$B_m = \sum_{j=1}^{J} A_{mj} \exp\left[-i\,\beta_0 k_m \{x \cos(\theta - \theta_x) + y \cos(\theta - \theta_y)\}\right] \tag{151}$$

for sea surface elevations, and by:

$$B_m = \sum_{j=1}^{J} A_{mj} T_m H_j \frac{\{e^{-k_m z} + s_1 e^{-k_m(2d-z)}\}}{\{1 + s_2 e^{-k_m d}\}}$$

$$* \exp\left[-i\,\beta_0 k_m \{x \cos(\theta - \theta_x) + y \cos(\theta - \theta_y)\}\right] \tag{152}$$

for the other wave properties. These can be expressed as a sum of products of separate functions of x, y, and z as follows:

1. Sea Surface

$$B_m = \sum_{j=1}^{J} A_{mj} \exp\left[i\,\beta_0 k_m x \cos(\theta - \theta_x)\right] \exp\left[i\,\beta_0 k_m y \cos(\theta - \theta_y)\right] \tag{153}$$

2. Other Wave Properties

$$B_m = \sum_{j=1}^{J} \frac{A_{mj} T_m H_j}{1 + s_2 e^{-k_m d}} \left\{ e^{-k_m z} + s_1 e^{-k_m (2d - z)} \right\}$$

$$* \exp\left[-i \beta_0 k_m x \cos(\theta - \theta_x)\right] \exp\left[-i \beta_0 k_m y \cos(\theta - \theta_y)\right] \tag{154}$$

Suppose it is desired to obtain a good representation locally in the vicinity of the structure. Consider the volume,

$$x_0 - D_1 \leq x \leq x_0 + D_1 \tag{155}$$

$$y_0 - D_2 \leq y \leq y_0 + D_2 \tag{156}$$

$$0 \leq z \leq d \tag{157}$$

(Here z has been taken as positive downward and zero at mean water level.)

It is natural to scale function as follows:

1. Sea Surface

$$B_m = \sum_{j=1}^{J} A_{mj} \exp\left[-i \beta_0 k_m x_0 \cos(\theta - \theta_x)\right] \exp\left[-i \beta_0 k_m D_1 \left(\frac{x - x_0}{D_1}\right) \cos(\theta - \theta_x)\right]$$

$$* \exp\left[-i \beta_0 k_m y_0 \cos(\theta - \theta_y)\right] \exp\left[-i \beta_0 k_m D_2 \left(\frac{y - y_0}{D_2}\right) \cos(\theta - \theta_y)\right]$$

..... (158)

2. Other Wave Properties

$$B_m = \sum_{j=1}^{J} \frac{A_{mj} T_m H_j}{1 + s_2 e^{-k_m d}} \left\{ e^{-k_m z} + s_1 e^{-k_m(2d-z)} \right\}$$

$$* \exp\left[-i\,\beta_0 k_m x_0 \cos(\theta - \theta_x)\right] \exp\left[-i\,\beta_0 k_m D_1 \left(\frac{x - x_0}{D_1}\right) \cos(\theta - \theta_x)\right]$$

$$* \exp\left[-i\,\beta_0 k_m y_0 \cos(\theta - \theta_y)\right] \exp\left[-i\,\beta_0 k_m D_2 \left(\frac{y - y_0}{D_2}\right) \cos(\theta - \theta_y)\right]$$

..... (159)

Let

$$\exp\left[-1\beta_0 k_m D_1 \left(\frac{x - x_o}{D_1}\right) \cos(\theta - \theta_x)\right] = \sum_{\alpha=0}^{N} \tilde{a}_\alpha p_\alpha \left(\frac{x - x_0}{D_1}\right) \qquad (160)$$

$$\exp\left[-i\beta_0 k_m D_2 \left(\frac{y - y_o}{D_2}\right) \cos(\theta - \theta_y)\right] = \sum_{\beta=0}^{N} \tilde{b}_\beta p_\beta \left(\frac{y - y_0}{D_2}\right) \qquad (161)$$

$$e^{-k_m z} = \sum_{\gamma=0}^{N} \tilde{c}_\gamma p_\gamma^*\left(e^{-k_0 z}\right) \qquad (162)$$

where k_0 is a selected single reference wave number. For many applications, the second term,

$$e^{-k_m(2d-z)} \qquad (163)$$

is negligible because depth is large. For the moment, suppose that this second term can be ignored (it will be reintroduced later). Then, if

$$\begin{aligned} u &= (x - x_0)/D_1 \\ v &= (y - y_0)/D_2 \\ w_1 &= e^{-k_0 z} \end{aligned} \qquad (164)$$

we have the following:

1. Sea Surface

$$B_m = \sum_{j=1}^{J} A_{mj} \exp\{-i\,\beta_0 k_m[x_0 \cos(\theta - \theta_x) + y_0 \cos(\theta - \theta_y)]\}$$

$$* \left[\sum_{\alpha=0}^{N} \tilde{a}_\alpha\, p_\alpha(u)\right]\left[\sum_{\beta=0}^{N} \tilde{b}_\beta\, p_\beta(v)\right] \tag{165}$$

2. Other Wave Properties

$$B_m = \sum_{j=1}^{J} \frac{A_{mj} T_m H_j \exp\{-i\,\beta_0 k_m[x_0 \cos(\theta - \theta_x) + y_0 \cos(\theta - \theta_y)]\}}{1 + s_2 e^{-2k_m d}}$$

$$* \left[\sum_{\alpha=0}^{N} \tilde{a}_\alpha\, p_\alpha(u)\right]\left[\sum_{\beta=0}^{N} \tilde{b}_\beta\, p_\beta(v)\right]\left[\sum_{\gamma=0}^{n} \tilde{c}_\gamma\, p_\gamma^*\left(e^{-k_0 z}\right)\right] \tag{166}$$

If these are substituted into Equation 150:

$$b(n\,\Delta t) = \sum_{\alpha=0}^{N} \sum_{\beta=0}^{N} \sum_{\gamma=0}^{N} Q_{\alpha,\beta,\gamma}(n\,\Delta t)\, p_\alpha(u)\, p_\beta(v)\, p_\gamma^*(w_1) \tag{167}$$

where (sea surface case):

$$Q_{\alpha,\beta,\gamma}(n\,\Delta t) = \sum_{m=0}^{N-1} \left[\sum_{j=1}^{J} \tilde{a}_\alpha \tilde{b}_\beta A_{mj} \exp\{-i\,\beta_0 k_m x_0 \cos(\theta - \theta_x)\}\right.$$

$$\left. * \exp\{-i\,\beta_0 k_m y_0 \cos(\theta - \theta_y)\}\right] e^{i2\pi mn/N} \tag{168}$$

Equation 168 is an FFT of the quantity within [] for each combination of α, β, and γ.

The other wave properties have a similar expression, with

$$Q_{\alpha,\beta,\gamma}(n\,\Delta t) = \sum_{m=0}^{N-1}\left[\sum_{j=1}^{J} \tilde{a}_\alpha \tilde{b}_\beta \tilde{c}_\gamma A_{mj} T_m H_j \right.$$
$$\left. * \frac{\exp\left\{-i\,\beta_0 k_m\left[x_0 \cos(\theta-\theta_x) + y_0 \cos(\theta-\theta_y)\right]\right\}}{1 + s_2 e^{-2k_m d}}\right] \tag{169}$$

$$e^{i2\pi mn/N}$$

At a given time, $n\Delta t$, many of the $Q_{\alpha,\beta,\gamma}$ for a given wave property are negligible. After all, the region $x_0 \pm D_1$ and $y_0 \pm D_2$ is relatively small compared to the wavelengths. Hence, low order polynomials are all that are required in order to represent the variation over the horizontal region. The vertical variation is attenuated more or less exponentially with depth, so a polynomial in w_1 should only need relatively low order.

Hence, at a given time, only a few $Q_{\alpha,\beta,\gamma}$ will be needed to represent the wave property. The particular coefficient needed may, however, be different from one time step to another. An interesting point here is that the $Q_{\alpha,\beta,\gamma}$ $(n\Delta t)$ can be, themselves, computed by the FFT algorithm simultaneously for $n = 0,1,2,\ldots,N\text{-}1$.

Up to this point, the actual computation of $\tilde{a}_\alpha$, $\tilde{b}_\beta$ *and* $\tilde{c}_\gamma$ *has not been explicitly* stated. From the definition of orthogonal polynomials,

$$\tilde{a}_\alpha = \frac{2\alpha+1}{2}\int_{-1}^{1} \exp\left[-i\,k_m \beta_0 D_1 \cos(\theta-\theta_x)_u\right] p_\alpha(u)\,du \tag{170}$$

$$\tilde{b}_\beta = \frac{2\beta+1}{2}\int_{-1}^{1} \exp\left[-i\,k_m \beta_0 D_2 \cos(\theta-\theta_y)_v\right] p_\beta(v)\,dv \tag{171}$$

$$\tilde{c}_\gamma = (2\gamma+1)\int_{0}^{1} w_1^{k_m/k_0}\, p_\gamma^*(w_1)\,dw_1 \tag{172}$$

Other depth term

Let

$$w_2 = e^{-k_0(2d - z)} \tag{173}$$

Then an exactly similar expansion with the same coefficients can be developed. The resulting representation of the wave property is:

$$b(n\,\Delta t) = \sum_{\alpha=0}^{N} \sum_{\beta=0}^{N} \sum_{\gamma=0}^{N} Q_{\alpha,\beta,\gamma}(n\,\Delta t)\, p_\alpha(u)\, p_\beta(v) \left\{ p_\gamma^*(w_1) + s_1\, p_\gamma^*(w_2) \right\} \tag{174}$$

Statistics of Random Wave Geometry

In testing the statistics of wave properties, such as height and period, it is important to distinguish between properties for individual waves and average properties above a specified time interval. The individual wave properties are the properties that a single wave possesses at a particular instant in time. Individual wave heights are usually defined in terms of maxima or minima between zero crossings of the water level elevation above mean water level or in terms of a time-varying envelope function. Individual wave periods may be defined in terms of time interval between up-crossings or in terms of functions of time derivatives of various orders of the water level elevation. In contrast, time interval average properties such as significant wave height or predominant wave period are usually taken from the wave spectral density holding during that time interval.

There is a very extensive literature on statistics of wave properties. Review and expository papers are given by Borgman (1982a) and Ochi (1982). In addition, there are expository chapters in recent books (Goda, 1985, Chapter 9; Muga, 1984; and Sarpkaya and Isaacson, 1981). The reader is referred to the above for an entry into the literature. The comments here will be directed to more recent results or to topics not generally covered in the cited references.

The usual choice for the probability law for individual wave heights is the Rayleigh distribution, or some closely related generalization, such as the Weibull distribution. If the significant wave height parameter, H_S, in the Rayleigh distribution is determined from the traditional 4.004 times the area under the wave spectral density (Borgman, 1982a, Equation 19), then empirical data on wave heights usually show a deviation from the Rayleigh probabilities for large waves (Forristall, 1978; Krogstad, 1985). An appropriate modification of the Rayleigh formula that agrees well with data is a form of the Weibull distribution:

$$F_H(h) = P[H \le h] = 1 - \exp\left[-(4h/H_s)^\alpha/\beta\right] \tag{175}$$

If $\alpha = 2$ and $\beta = 8$, then the above formula reduces to the usual Rayleigh distribution function. Forristall found that $\alpha = 2.126$ and $\beta = 8.42$ were good choices for his data. Krogstad reports values of α ranging from 2.37 to 2.50 and values of β from 12.5 to 15.6.

An alternative to the Weibull modification is suggested by Goda (1978), who found that the significant wave height is better estimated by using 3.8 times the area under the spectral density instead of the narrow-band theory value of 4.004. Similarly, Chen (1979, p. 19) found the best multiplier to be 3.88 for hurricane Carla and 3.82 for hurricane Camille (both Gulf of Mexico storms). Horikawa (1988, p. 46) discusses some of these results. With these reduced values of H_s, the Rayleigh distribution gives fair results without modification. Good results with the Rayleigh formula are also obtained when the mean-square (zero-crossing) wave heights can be obtained directly from water level elevation time series (Borgman, 1973b, Figure 7).

The conditional probability law for individual wave periods, given the wave height, was shown by Longuet-Higgins (1975) to be normally distributed for waves with a narrow-band spectral density. This result was shown by Chen (1979) and Chen, Borgman, and Yfantis (1979) to hold approximately for hurricane waves for wave heights larger than the significant wave height. In particular, Chen found that $(T|H = h)$ was approximately normal with the mean equal to $0.85T_p$ and the standard deviation equal to $0.15T_pH_s/h$. Here, T_p is the period associated with the frequency at the peak or mode of the wave spectral density and H_s is the significant wave height. Chen's results were based on an analysis of wave records in hurricane Carla and Camille in the Gulf of Mexico. Krogstad (1985) found similar results for North Sea waves although the standard deviations were somewhat larger. Krogstad suggests that this may be due to the presence of swell in his data. Goda (1978) found quite similar results in a suite of data that he had collected.

The joint probability law of height and direction of travel for individual waves was developed by Borgman (1981) and later extended to treat the joint probability law for height, period, and direction (Ogbi, 1983). The probability law for height and direction appears to agree well in the data comparisons that have been made, while the height-period-direction trivariate law is much more speculative and needs to be checked more thoroughly against data.

Probability laws for wave properties over a period of time, such as the maximum wave height, have received substantial attention in the literature. In the multiyear or decade frame of reference, probabilities for maximum wave height become intimately interrelated to the storm climatology of the region. A very detailed study of such topics for the Gulf of Mexico was sponsored by a "consortium" of oil companies (Ward, Borgman, and Cardone, 1979). Other discussions of the probability law for long-term maximum wave height are given by Ochi (1982, p. 283ff.) and Naess (1984). The probabilities for maximum wave height over a time interval within which the random seas are statistically stationary may be treated with extensions from the Rayleigh distribution (Borgman, 1973b; Ochi, 1982, p. 287ff.; Goda, 1985, p. 227) or from stochastic process theory (Boccotti, 1985; Naess, 1985).

The study of the long-term probabilities for spectral shape parameters is very data intensive and specific for each location. Usually, this kind of study involves a cooperative effort between meteorologists for the wave hindcasts, engineers for relevant structural concerns, and statisticians for the probabilistic data analysis (Ward, Borgman, and Cardone, 1979; Borgman and Resio, 1982),

Wave Group Simulation

The statistics of wave groups is really a topic in its own right, with rather different mathematical techniques and concepts (Goda, 1985, p. 230ff.), and will not be treated here. Instead, several comments will be introduced regarding ways that wave groups can be embedded into a conditional simulation. If a short time history of wave group water level elevations is available from measurements, the rest of the record can be conditionally simulated on reproducing that interval of data within the simulation. Thus, one can produce a number of simulations, each containing exactly the specified group.

Another way to force the occurrence of a wave group, within exactly specifying the time history, is to introduce a short sequence of large water level elevations that are separated approximately by the period of the waves in the group. That is, each large value is placed in the time record at a separation of, say, 10 seconds from the previous and subsequent large values. The intermediate time series values and the rest of the wave record are then conditionally simulated, given the assigned magnitudes of the large values. The simulated wave amplitudes in the group will be at least as large as the large values, but some of the simulated intermediate values may rise a little height. This forces the period of the waves in the simulated group to vary sightly from the interval of separation of the imposed large values and allows the wave heights in the group to vary somewhat from simulation to simulation.

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LIST OF SYMBOLS

a	Parameter in von Mises spreading function in Equation 65
a	Wave amplitude in Equation 2
a_n, a_n*	Multiplying coefficients for orthogonal polynomials in the representation of an arbitrary function in Equation 143s, 147, and 148
$a_x(x,y,z,t)$	x component of water particle acceleration
$a_y(x,y,z,t)$	y component of water particle acceleration
$a_z(x,y,z,t)$	z component of water particle acceleration
$\tilde{a}_\alpha$	Multiplier coefficient for Legendre polynomials in Equation 160, defined in Equation 170
A_{mj}	Complex-valued wave amplitude at frequency index m and direction index j (see Equation 39)
b_n, b_n^*	Coefficients in orthogonal polynomials in Equation 133
$b(x,y,z,t)$	General representation of a wave property in Equations 11 and 23
$B(f;x,y,z)$	Fourier transform of (x,y,z) (see Equation 25)
$b(n\Delta t)$	Time sequence of general wave properties
$B(m\Delta f)$	Frequency sequence of FFT of general wave property
$\tilde{b}_\beta$	Multiplier coefficient for Legendre polynomials in Equation 161, defined in Equation 171
c	Parameter in wrapped normal spreading function in Equation 63
c_k	Sequence of covariance in Equation 128
$\tilde{c}_\gamma$	Multiplier coefficient for shifted Legendre polynomials in Equation 131, defined in Equation 172
C	Covariance matrix of **V** in Equation 89
C^{-1}	Inverse of matrix C
C^{+}	Generalized inverse of matrix C
$\|C\|$	Determinant of matrix C
d	Water depth
d_Δ	Parameter in delta stretching in Equation 74

$D(\theta; f)$	Direction spreading function in Equation 43
D_1	Half-width of x variation in Equation 155
D_2	Half-width of y variation in Equation 156
$E[.]$	Statistical expectation operator
f	Frequency
f_0	Frequency at mode of spectral density in Equation 46
$f_R(r)$	Rayleigh probability density in Theorem A
$f_{X,Y}(x,y)$	Probability density in Theorem A
$F(a)$	Rayleigh distribution function in Equation 81
$F_V(v)$	Multivariate distribution function for random vector **V**, with argument **v**, in Equation 89
g	Acceleration due to gravity
$g(x)$	Arbitrary function of x in Equation 143
$G(z)$	Function in general representation of a wave property
$G(\lambda)$	Function of λ in Equation 50
$H(\sigma)$	General function in the representation of a wave property = $\sqrt{-1}$
j	Direction index
J	Number of angle increments to cover circle of directions at angle increment $\Delta\theta$
J	Jacobian of transformation in Theorem A
k	Wave number = 2π/wave length
k	Lag index in Equation 128
k_m	Wave number in Equation 149 for frequency index m
k_0	Single reference wave number in Equation 162
K	Norming constant in the generalized cosine-square spreading function in Equation 67
m	Sequence index in frequency series for the discrete Fourier transform in Equations 27 and 28
m_B	Beginning index for the interval of energetic in the frequencies in the frequency sequence (see Equation 36)

m_L	Last index for the interval of energetic in the frequency sequence (see Equation 36)
n	Sequence index in time series for the discrete Fourier transform in Equations 27 and 28
n	Index in Fourier series in Equation 63
n	Order of orthogonal polynomial in Equation 133
N	Length of time or frequency sequences in the definition of the discrete Fourier transform in Equations 27 and 28
$N(\mu, C)$	Designation of a normal probability law with mean vector μ and covariance matrix C
p(x,y,z,t)	Water pressure deviation from static pressure
$p_n(x)$	Legendre polynomial in Equation 133
$p_n^*(x)$	Shifted Legendre polynomial in Equation 133
P	Parameter in Ochi-Hubble spectral model in Equation 50
Q	Integral square error of approximation in Equation 144
$Q_{mj}^{(\lambda)}$	Combination of functions in Equation 102
$Q_{\alpha,\beta,\gamma}(n\,\Delta t)$	Total coefficient in multivariate orthogonal polynomial expansion of the wave property at time $n\Delta t$ in Equation 169
$R = \sqrt{X^2 + Y^2}$	In Theorem A
s_1	Integer in Equation 149 = -1 for sinh() numerator wave property in G(z) = +1 for cosh() numerator wave property in G(z)
s_2	Integer in Equation 149 = -1 for sinh() numerator wave property in G(z) = +1 for cosh() numerator wave property in G(z)
S(f)	Frequency spectral density
$S(f,\theta)$	Direction spectral density in Equation 40
S_0	Magnitude of S(f) at mode of spectral density in Equation 49
t	Time coordinate

$T(f)$	Function in general representation of a wave property
u	Dimensionless x position in Equation 164
U	Uniform random number in Equation 83
U_{mj}	Real part of A_{mj}
v	Dimensionless y position in Equation 164
$V_x(x,y,z,t)$	x component of water particle velocity
$V_y(x,y,z,t)$	y component of water particle velocity
$V_z(x,y,z,t)$	z component of water particle velocity
$(\mathbf{V}_2 \mid \mathbf{V}_1 = \mathbf{v}_1)$	Conditional random vector $\mathbf{V}_2$, given that random vector $\mathbf{V}_1$ is equal to $\mathbf{v}_1$
V_{mj}	Negative of imaginary part of A_{mj}
w_n	General time sequence in definition of the discrete Fourier transform in Equation 28
W_m	General frequency sequence in definition of the discrete Fourier transform in Equation 27
w_1	Dimensionless z position function in Equation 164
w_2	Dimensionless z position function in Equation 173
x	Horizontal coordinate of position
x	Argument of orthogonal polynomial in Equation 133
X	Random variable in Theorem A
y	Horizontal coordinate of position
y	Variable in integration change-of-variable in Equations 47 and 48
Y	Random variable in Theorem A
z	Vertical coordinate of position
z_Δ	Delta-stretched z value in Equation 74
z_s	Stretched z value in Equation 72
Z	Standard normal random number in Equation 86
α	General angle in Equations 9 and 10
α	Peakedness parameter in the generalized cosine-squared spreading function in Equation 67

α	Parameter in gamma stretching in Equation 77
β_0	= 1 if the waves are traveling toward direction θ
	= -1 if waves are coming from direction θ
β	Parameter in gamma stretching in Equation 77
γ_n	Time sequence in Theorem C
Γ_m	Discrete Fourier transform of γ_n in Theorem C
$\Gamma(\lambda)$	Complete gamma function
δ	Effective width in Equation 51
Δ	Parameter in delta stretching in Equation 74
Δf	Frequency increment (see Equations 31 and 32)
Δt	Time increment (see Equations 31 and 32)
$\Delta\theta$	Angle increment
ϵ_1,ϵ_2	Expectations in Theorem B
$\eta(x,y,t)$	Water level elevation
θ	Direction of wave travel
θ_x	Direction of position x axis
θ_y	Direction of position y axis
θ_{HP}	Half-peak angular spread in Equations 64, 66, and 68
θ_0	Principle direction of spreading function in Equations 63, 65, and 67
λ	Peakedness parameter in the Ochi-Hubble spectral model in Equation 46
λ	Subscript in Equations 99 and 100
μ	Mean vector of random variable **V** in Equation 89
ν	Maximum lag of importance in covariance sequence in Equation 128, that is, $c_k = 0$ if $k > \nu$
ρ	Water density
ρ_0	Wave property at z1 = 0 in Equation 71
ρ_0'	Derivative of wave property with respect to z at z = 0 in Equation 71
$\rho_s(z)$	Stretched wave property at elevation z in Equation 77
σ^2	Variance of sea surface in Equation 46
σ_η	Standard deviation of sea surface elevation in Equation 76
Φ, ϕ	Wave phase
Φ_m	Real part of $B(m\Delta f)$ in Equation 106
Φ	= arctan(Y/X) in Theorem A
Ψ_m	Imaginary part of $B(m\Delta f)$ in Equation 106

∞	Infinity
subscript T	Matrix transpose
subscript *	Conjugation

Appendix

THEOREMS

THEOREM A

Let (X, Y) be random variables with mean zero and variance σ^2. Define

$$R = (X^2 + Y^2)^{1/2}$$

$$\Phi = \arctan(Y/X)$$

Then X and Y are independent and normally distributed if and only if R and ϕ are independent. R is Rayleigh distributed, and ϕ is uniformly distributed on $(0, 2\pi)$.

Note: The Rayleigh distribution here has probability density

$$f_R(r) = \begin{cases} (r/\sigma^2), & \exp(-r^2/2\sigma^2), \quad r \geq 0 \\ 0, & \text{for } r < 0 \end{cases}$$

Proof. Let X and Y be $N(0, \sigma^2)$ and independent,

$$f_{X,Y}(x, y) = (1/2\pi\sigma^2)\exp\left[-(x^2 + y^2)/2\sigma^2\right]$$

Let

$$X = R\cos\Phi$$

$$Y = R\sin\Phi$$

or

$$R = \left(x^2 + y^2\right)^{1/2}$$

$$\Phi = \arctan(Y/X)$$

Then the Jacobian of the transformation is

$$J = \frac{\partial(x,y)}{\partial(r,\phi)} = \begin{bmatrix} \cos\phi & -r\sin\phi \\ \sin\phi & r\cos\phi \end{bmatrix} = r$$

Hence,

$$f_{R,\Phi}(r,\phi) = (1/2\,\pi\,\sigma^2)\exp(-r^2/2\,\sigma^2)(J) = (1/2\,\pi)(r/2\,\sigma^2)\exp(-r^2/2\,\sigma^2)$$

define for $0 < \phi < 2\pi$ and $r > 0$. But the density for a uniform random variable on $(0, 2\pi)$ is:

$$f_{\Phi}(\phi) = \begin{cases} 1/2\,\pi, & \text{for } 0 < \phi < 2\,\pi \\ 0, & \text{otherwise} \end{cases}$$

So

$$f_{R,\Phi}(r,\phi) = f_{\Phi}(\phi)\,f_R(r)$$

It follows that R and Φ are independent. Φ is uniform on $(0, 2\pi)$, and R is Rayleigh distributed.

THEOREM B (Borgman 1973a)

Let $i = \sqrt{-1}$ and $b_n = \sum_{m=0}^{N-1} B_m \exp(12\,\pi\,m\,n/N)$ for $n = 0,1,2,\ldots,N-1$, be a covariance-stationary, mean zero, sequence of real-valued random variables that are periodic with period N and a realization of a time series sampled with time increment Δt. That is,

$$E[b_n] = 0$$

and

$$C_{bb}(k) = E[b_n b_{n+k}]$$

does not depend on n. Then if

$$\Delta f = 1/(N \Delta t)$$

$$S_{BB}(m) = \Delta t \sum_{k=0}^{N-1} C_{bb}(k) \exp(-i 2 \pi m n/N)$$

and

$$B_m = U_m - i V_m$$

It follows that:

1. $V_0 = V_{N/2} = 0$:
2. for $0 < m < N/2$, $U_m = U_{N-m}$ and $V_m = -V_{N-m}$;
3. $E[U_m] = E[V_m] = 0$:
4. $C_{bb}(N-k) = C_{bb}(k)$ and $S_{BB}(N-m) = S_{BB}(m)$ are real-valued;
5. $\{U_0, U_1, V_1, U_2, V_2, \ldots, U_{(N/2)-1}, V_{(N/2)-1}, U_{N/2}\}$ are uncorrelated;
6. $Var[U_0] = S_{BB}(0)\Delta f$, $Var[U_{N/2}] = S_{BB}(N/2)\Delta f$, $Var[U_m] = Var[V_m] = S_{BB}\Delta f/2$, [for $m = 1,2,\ldots,(N/2)-1$], and $S_{BB}(m)\Delta f = E[|B_m|^2]$ for all m.

Proof. By the discrete Fourier transform inverse,

$$B_m = \frac{1}{N} \sum_{n=0}^{N-1} b_n \exp(-i 2 \pi m n/N)$$

Because b_n is real-valued.

$$B_m^* = \frac{1}{N}\sum_{n=0}^{N-1} b_n \exp(i\,2\,\pi\, m\, n/N)\exp(-i\,2\,\pi\, n\, N/N)$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} b_n \exp[-i\,2\,\pi\,(N-m)\,n/N]$$

$$= B_{N-m}$$

This proves items 1 and 2 when combined with the periodicity. Also, from the FFT inverse,

$$E[B_m] = \frac{1}{N}\sum_{n=0}^{N-1} E[b_n]\exp(-i\,2\,\pi\, m\, n/N) = 0$$

This proves item 3. Because,

$$\begin{aligned} C_{bb}(N-k) &= E[b_n\, b_{n+N-k}] = E[b_n\, b_{n-k}] && \text{by periodicity} \\ &= E[b_{n-k}\, b_n] = E[b_n\, b_{n+k}] && \text{by covariance stationarity} \\ &= C_{bb}(k) \end{aligned}$$

From the symmetry on $C_{bb}(k)$,

$$S_{BB}(m) = \Delta t \sum_{k=0}^{N-1} C_{bb}(k)\exp(2\,\pi\, k\, m/N)$$

which is real-valued. Thus, item 4 is proven.

For $0 \le m \le N/2$ and $0 \le m'2 \le N/2$, consider:

$$e_1 = E[B_m^* B_{m'}] = \{E[U_m U_{m'}] + E[V_m V_{m'}\} + i\{E[V_m U_{m'}] - E[U_m V_{m'}]\}$$

$$e_2 = E[B_m B_{m'}] = \{E[U_m U_{m'}] + E[V_m V_{m'}\} + i\{E[V_m U_{m'}] - E[U_m V_{m'}]\}$$

These two expectations can also be expressed in terms of b_n as

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E[b_n b_{n'}] \exp\left[-i 2\pi \begin{pmatrix} m'n' - mn \\ m'n' + mn \end{pmatrix} \Big/ N\right]$$

Now let $k = n' - n$, $E[b_n b_{n'}] = C_{bb}(n'-n) = C_{bb}(k)$, and using the periodicity after setting $n' = k+n$.

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{1}{N^2} \sum_{k=0}^{N-1} C_{bb}(k) \exp(-i 2\pi m'k/N) \sum_{n=0}^{N-1} \exp\left[-i 2\pi \begin{pmatrix} m' - m \\ m' + m \end{pmatrix} n \Big/ N\right]$$

But

$$\sum_{n=0}^{N-1} \exp\left[-i 2\pi \begin{pmatrix} m' - m \\ m' + m \end{pmatrix} n \Big/ N\right] = \begin{cases} \begin{pmatrix} N \\ 0 \end{pmatrix}, & \text{if } m = m' \\ \begin{pmatrix} 0 \\ N \end{pmatrix}, & \text{if } m = -m' \\ 0, & \text{otherwise} \end{cases}$$

With the stated constraints on m and m', m = m' is impossible. Hence, introducing the definition for S_m,

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{cases} \begin{pmatrix} S_{BB}(m)\,\Delta f \\ 0 \end{pmatrix}, & \text{if } m = m',\ m \neq 0,\ m \neq N/2 \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, & \text{if } m \neq m' \\ \begin{pmatrix} S_{BB}(m)\,\Delta f \\ S_{BB}(m)\,\Delta f \end{pmatrix}, & \text{if } m = m' = 0 \text{ or } m = m' = N/2 \end{cases}$$

If m = m', m ≠ 0, and m ≠ N/2.

$$E\left[U_m^2\right] + E\left[V_m^2\right] = S_{Bb}(m)\,\Delta f$$

$$E\left[U_m V_m\right] - E\left[U_m V_m\right] = 0$$

$$E\left[U_m^2\right] - E\left[V_m^2\right] = 0$$

$$E\left[U_m V_m\right] + E\left[U_m V_m\right] = 0$$

It follows that, for $0 < m < N/2$,

$$E\left[U_m^2\right] = E\left[V_m^2\right] = S_{BB}(m)\,\Delta f/2$$

$$E\left[U_m V_m\right] = 0$$

For m = 0 or m = N/2, $V_m = 0$ by item 1. So $E\left[U_0^2\right] = S_{BB}(0)\,\Delta f$ and $E\left[U_{N/2}^2\right] = S_{Bb}(0)\,\Delta f$.

If m ≠ m',

$$E\left[U_m U_{m'}\right] + E\left[V_m V_{m'}\right] = 0$$

$$E\left[V_m U_{m'}\right] - E\left[U_m V_{m'}\right] = 0$$

$$E\left[U_m U_{m'}\right] - E\left[V_m V_{m'}\right] = 0$$

$$E\left[V_m U_{m'}\right] + E\left[U_m V_{m'}\right] = 0$$

Therefore,

$$E[U_m U_{m'}] = E[V_m V_{m'}] = E[V_m U_{m'}] = E[U_m V_{m'}] = 0$$

Finally, if m = m',

$$E[|B_m|^2] = E[U_m^2 + V_m^2] = S_{BB}(m)\,\Delta f$$

This completes the proof of items 5 and 6.

THEOREM C (Borgman, 1973a; see Goodman, 1957 for continuous version).

Let the two sequences

$$b_n = \sum_{m=0}^{N-1} B_m \exp(i\,2\,\pi\,m\,n/N)$$

$$\gamma_n = \sum_{m=0}^{N-1} \Gamma_m \exp(i\,2\,\pi\,m\,n/N)$$

for n = 0,1,2,...,N-1, be covariance - and cross-covariance - stationary, have mean zero, be real-valued, be periodic with period N, and be a time series realization sampled at the time increment Δt. Define $\Delta f = 1/(N\Delta t)$ and

$$C_{bb}(k) = E[b_n b_{n+k}]$$

$$C_{\gamma\gamma}(k) = E[\gamma_n \gamma_{n+k}]$$

$$C_{b\gamma}(k) = E[b_n \gamma_{n+k}]$$

$$C_{\gamma b}(k) = E[\gamma_n b_{n+k}]$$

$$\begin{bmatrix} S_{BB}(m) & S_{B\Gamma}(m) \\ S_{\Gamma B}(m) & S_{\Gamma\Gamma}(m) \end{bmatrix} = \Delta t \sum_{k=0}^{N-1} \begin{bmatrix} C_{bb}(k) & C_{b\gamma}(k) \\ C_{\gamma b}(k) & C_{\gamma\gamma}(k) \end{bmatrix} \exp(-i\,2\,\pi\,m\,k/N)$$

$$S_{B\Gamma}(m) = c_{B\Gamma}(m) - i\,q_{B\Gamma}(m)$$

$$B_m = U_m - i\,V_m$$

$$\Gamma_m = \Phi_m - i\,\Psi_m$$

It follows that:

1. Both sequences have the individual properties in theorem B:
2. $C_{b\gamma}(k) = C_{\gamma b}(N-k)$ are real-valued;
3. $S_{B\Gamma}(m) = S^{*}_{B\Gamma}(N-m) = S_{\Gamma B}(N-m)$ are complex-valued;
4. for $0 \le m \le N/2$ and $0 \le m' \le N/2$ with $m \ne m'$, (U_m, V_m) are uncorrelated with $(\Phi_{m'}, \Psi_{m'})$
5. if $0 < m < N/2$.

$$\text{covariance matrix of} \begin{bmatrix} U_m \\ V_m \\ \Phi_m \\ \Psi_m \end{bmatrix} = \begin{bmatrix} S_{BB}(m) & 0 & c_{B\Gamma}(m) & q_{B\Gamma}(m) \\ 0 & S_{BB}(m) & -q_{B\Gamma}(m) & c_{B\Gamma}(m) \\ c_{B\Gamma}(m) & -q_{B\Gamma}(m) & S_{\Gamma\Gamma}(m) & 0 \\ q_{B\Gamma}(m) & c_{B\Gamma} & 0 & S_{\Gamma\Gamma} \end{bmatrix} \frac{\Delta f}{2}$$

if m = 0 or m = N/2,

$$\textit{covariance matrix of} \begin{bmatrix} U_m \\ \Phi_m \end{bmatrix} = \begin{bmatrix} S_{BB}(m) & c_{B\Gamma}(m) \\ c_{B\Gamma}(m) & S_{\Gamma\Gamma}(m) \end{bmatrix} \Delta f$$

$$S_{B\Gamma}(m)\, \Delta f = E\left[B_m^* \Gamma_m\right]$$

Proof. Item 1 is obviously true. Because,

$$\begin{aligned} C_{\gamma b}(N-k) &= E\left[\gamma_n b_{n+N-k}\right] \text{ by periodicity} \\ &= E\left[b_{n-k} \gamma_n\right] \\ &= E\left[b_n \gamma_{n+k}\right] \text{ by covariance-ctationarity} \\ &= C_{b\gamma}(k) \end{aligned}$$

This proves item 2 since the real-value property follows directly from the definition. Continuing,

$$\begin{aligned} S_{B\Gamma}^*(N-m) &= \Delta t \sum_{k=0}^{N-1} C_{b\gamma}(k) \exp[+i 2 \pi (N-m) k/N] \\ &= \Delta t \sum_{k=0}^{N-1} C_{b\gamma}(k) \exp(-12 \pi\, m\, k/N) \\ &= S_{B\Gamma}(m) \\ &= \Delta t \sum_{k=0}^{N-1} C_{\gamma b}(N-k) \exp[+i 2 \pi m (N-k)/N] \end{aligned}$$

Let j = N-k and use the periodicity $C_{\gamma b}(N)$. Then,

$$S^*_{B\Gamma}(N-m) = \Delta t \sum_{k=0}^{N-1} C_{\gamma b}(j) \exp[i\, 2\, \pi\, m\, j/N]$$

$$= S^*_{\Gamma B}(m)$$

This proves item 3.

The proof of item 4, 5 and 6 is very parallel to the corresponding proof in Theorem B of item 5 and 6. Let

$$e_1 = E\left[B^*_m \Gamma_{m'}\right]$$

$$e_2 = E\left[B_m \Gamma_{m'}\right]$$

and perform the parallel algebra. This gives:

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{cases} \begin{pmatrix} S_{B\Gamma}(m)\,\Delta f \\ 0 \end{pmatrix}, & \text{if } m = m',\ m \neq 0,\ m \neq N/2 \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, & \text{if } m \neq m' \\ \begin{pmatrix} S_{B\Gamma}(m)\,\Delta f \\ S_{B\Gamma}(m)\,\Delta f \end{pmatrix}, & \text{if } m = m' = 0 \text{ or } m = m' = N/2 \end{cases}$$

The proof of item 4 and 5 follows from the same steps used in Theorem B.

THEOREM D

For $\lambda = 1, 2$, define:

$$b_n^{(\lambda)} = \sum_{m=0}^{N-1} \sum_{j=1}^{J} A_{mj} Q_{mj}^{(\lambda)} \exp(i\, 2\, \pi\, m\, n/N)$$

$$B_m^{(\lambda)} = \sum_{j=1}^{J} A_{mj} Q_{mj}^{(\lambda)} = \Phi_m^{(\lambda)} + i\, \Psi_m^{(\lambda)}$$

$$A_{mj} = U_{mj} - i\, V_{mj}$$

$$Q_{mj}^{(\lambda)} = Q_{N-m,j}^{(\lambda)*}$$

where $\{(U_{mj}, V_{mj}),\ m = 0,1,2\ldots,N/2;\ j = 1,2,\ldots,J\}$ are independent random variables with

$$V_{0j} = V_{N/2,j} \equiv 0$$

$$A_{mj}^{*} = A_{N-m,j}$$

$$\mathrm{Var}(U_{mj}) = \begin{cases} S_{mj}\, \Delta f\, \Delta\theta\,, & \text{if } m = 0 \text{ or } m = N/2 \\ S_{mj}\, \Delta f\, \Delta\theta/2\,, & \text{if } 1 \le m \le N/2 \end{cases}$$

$$\mathrm{Var}(V_{mj}) = S_{mj}\, \Delta f\, \Delta\theta/2$$

Here, $b_n^{(\lambda)}$ is interpreted as being realization of a time series sampled with time increment Δt, and Δf is defined as:

$$\Delta f = 1/(N\, \Delta t)$$

Then the $b_n^{(\lambda)}$ are real-valued, and the covariance relations for $b_n^{(1)}$, $b_n^{(2)}$, U_{mj}, and V_{mj} are as follows:

For $\lambda = 1, 2$:

1. $$\mathrm{Cov}\left[b_n^{(\lambda)}, b_{n'}^{(\lambda)}\right] = \Delta f\,\Delta\theta \sum_{m=0}^{N-1} \sum_{j=1}^{J} S_{mj} \left|Q_{mj}^{(\lambda)}\right|^2 \exp[-i\,2\,\pi\,m(n'-n)/N]$$

2. $$\mathrm{Cov}\left[b_n^{(1)}, b_{n'}^{(2)}\right] = \Delta f\,\Delta\theta \sum_{m=0}^{N-1} \sum_{j=1}^{J} S_{mj}\, Q_{mj}^{(1)*}\, Q_{mj}^{(2)} \exp[-i\,2\,\pi\,m(n'-n)/N]$$

If $0 < m < N/2$:

3. $$\mathrm{Cov}\left[U_{mj}, b_n^{(\lambda)}\right] = S_{mj}\,\Delta f\,\Delta\theta\,\mathrm{Re}\left[Q_{mj}^{(\lambda)} \exp(i\,2\,\pi\,m\,n/N)\right]$$

4. $$\mathrm{Cov}\left[V_{mj}, b_n^{(\lambda)}\right] = S_{mj}\,\Delta f\,\Delta\theta\,\mathrm{Im}\left[Q_{mj}^{(\lambda)} \exp(2\,\pi\,m\,n/N)\right]$$

If $m = 0$ or $m = N/2$:

5. $$\mathrm{Cov}\left[U_{mj}, b_n^{(\lambda)}\right] = 2\,S_{mj}\,\Delta f\,\Delta\theta\,\mathrm{Re}\left[Q_{mj}^{(\lambda)} \exp(i\,2\,\pi\,m\,n/N)\right]$$

6. If $0 < m < N/2$:

$$\mathrm{Cov}\left[U_{mj}, \Phi_m^{(\lambda)}\right] = -\mathrm{Cov}\left[V_{mj}, \Psi_m^{(\lambda)}\right] = S_{mj}\,\mathrm{Re}\left[Q_{mj}^{(\lambda)}\right] \Delta f\,\Delta\theta/2$$

$$\mathrm{Cov}\left[V_{mj}, \Phi_m^{(\lambda)}\right] = \mathrm{Cov}\left[U_{mj}, \Psi_m^{(\lambda)}\right] = S_{mj}\,\mathrm{Im}\left[Q_{mj}^{(\lambda)}\right] \Delta f\,\Delta\theta/2$$

$$\mathrm{Var}\left[\Phi_m^{(\lambda)}\right] = \mathrm{Var}\left[\Psi_m^{(\lambda)}\right] = \sum_{j=1}^{J} S_{mj} \left|Q_{mj}^{(\lambda)}\right|^2 \Delta f\,\Delta\theta/2$$

$$\mathrm{Var}\left[U_{mj}\right] = \mathrm{Var}\left[V_{mj}\right] = S_{mj}\,\Delta f\,\Delta\theta/2$$

$$\mathrm{Cov}\left[\Phi_m^{(\lambda)}, \Psi_m^{(\lambda)}\right] = 0$$

7. If $m = 0$ or $m = N/2$, $Q_{mj}^{(\lambda)}$ must be real-valued and:

$$\mathrm{Cov}\left[U_{mj}\,\Phi_m^{(\lambda)}\right] = S_{mj}\,Q_{mj}\,\Delta f\,\Delta\theta$$

$$\mathrm{Var}\left[U_{mj}\right] = S_{mj}\,\Delta f\,\Delta\theta$$

$$\mathrm{Var}\left[\Phi_m^{(\lambda)}\right] = \sum_{j=1}^{J} S_{mj}\left[Q_{mj}^{(\lambda)}\right]^2 \Delta f\,\Delta\theta$$

8. If $0 < m < N/2$:

$$\mathrm{Cov}\left[\Phi_m^{(1)}, \Phi_m^{(2)}\right] = \mathrm{Cov}\left[\Psi_m^{(1)}, \Psi_m^{(2)}\right] = \sum_{j=1}^{J} S_{mj}\,\mathrm{Re}\left[Q_{mj}^{(1)*}\,Q_{mj}^{(2)}\right]\Delta f\,\Delta\theta/2$$

$$\mathrm{Cov}\left[\Phi_m^{(1)}, \Psi_m^{(2)}\right] = -\mathrm{Cov}\left[\Psi_m^{(1)}, \Phi_m^{(2)}\right] = \sum_{j=1}^{J} S_{mj}\,\mathrm{Im}\left[Q_{mj}^{(1)*}\,Q_{mj}^{(2)}\right]\Delta f\,\Delta\theta/2$$

9. If $m = 0$ or $m = N/2$, $Q_m^{(\lambda)}$ is real-valued and

$$\mathrm{Cov}\left[\Phi_m^{(1)}, \Phi_m^{(2)}\right] = \sum_{j=1}^{J} S_{mj}\,Q_m^{(1)}\,Q_m^{(2)}\,\Delta f\,\Delta\theta$$

Proof. The proof just involves a careful application of the previous theorems.

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